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## Evolution of Close Binary Systems

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### ABSTRACT

We collect data on the masses, radii, *etc.* of three classes of close binary stars: low-temperature contact binaries (LTCBs), near-contact binaries (NCBs), and detached close binaries (DCBs). We restrict ourselves to systems where (i) both components are, at least arguably, near the Main Sequence, (ii) the periods are less than a day, and (iii) there is both spectroscopic and photometric analysis leading to reasonably reliable data. We discuss the possible evolutionary connections between these three classes, emphasising the roles played by mass loss and angular momentum loss in rapidly-rotating cool stars.

*Subject headings:* stars: binaries, eclipsing — stars: fundamental parameters, individual — stars: contact binaries — stars: near-contact binaries

### 1. Introduction

Since fundamental stellar parameters such as masses, radii, luminosities and chemical compositions provide us with information about stellar evolution, stars with well-defined parameters are especially important for more accurate knowledge about evolution. To determine the components' parameters accurately it is important to have both spectroscopic and photometric observations of binary systems. In this study we consider three selections of short-period binaries where there is high-quality spectroscopic and photometric data, and endeavour to understand possible evolutionary connections between these selections.

In order to use nomenclature in a consistent manner, let us mention the following concepts. First of all, eclipsing binary stars were classified according to their light-curves as Algol (EA),  $\beta$  Lyrae (EB) and W UMa (EW) types. Since this classification is based only

on the light-curve shape it is not very useful, and another classification which depends on the Roche geometry is to be preferred. According to this classification, if neither component fills its Roche lobe the binary is called ‘detached’ or D, whereas if one (and only one) of the component fills its lobe the binary is called ‘semi-detached’ or SD; finally if both components fill (or overfill) their lobes the binary is described as ‘contact’ or C. Although Algol ( $\beta$  Per) is semidetached, and is the prototype of EA systems, many EA systems have turned out to be detached. It is probably conventional nowadays to apply the description ‘Algol’ to SD binaries only, and furthermore to only those SDs in which the *less* massive component fills its Roche lobe, as in Algol itself. There do however exist SD systems (see Table 2) in which the *more* massive component fills its Roche lobe. We refer here to systems like Algol as SD2 systems, and to the others as SD1 systems, or ‘reverse Algols’.

In the first three Sections of this paper, concerning observational data, we take component 1 to be *always* the more massive component. This is by no means standard, but it seems a reasonable definition here since we concentrate almost exclusively on double-lined spectroscopic binaries, for which the mass ratio can be determined directly. We also take the mass ratio  $q$  to be  $q \equiv M_2/M_1 < 1$ . Again this is not standard. In the last three Sections, we consider the long-term evolution of binaries, where Roche-lobe overflow (RLOF) can alter the mass ratio so that it might increase to unity and above. In these Sections we take component 1 to be the *initially* more massive star. This may appear confusing, but unfortunately there is no prescription which is not potentially confusing.

The term ‘close binary’ commonly means a system where the separation of the components is small, which implies that tidal force and RLOF play important roles in their evolution. In this paper we consider the structure and evolution of three subsets of close binaries. We restrict ‘close’ to mean periods (somewhat arbitrarily) of less than a day. We exclude highly-evolved systems, those containing black holes, neutron stars, white dwarfs and hot (SDB, SDO) subdwarfs; we also exclude normal OB stars, although there are not many at these short periods. So the systems on which we concentrate contain stars both of which are at least arguably on or fairly near the main sequence, and  $\sim$  A0 or later. The three subsets that we discuss are (i) low-temperature contact binaries (LTCBs), (ii) semidetached binaries, which at such short periods are necessarily close to contact and so are described as near-contact binaries (NCBs), and (iii) detached close binaries (DCBs). We identify these alternatively as C, SD (SD1 or SD2) and D, respectively. We expect strong evolutionary connections between these classes, and we attempt to determine to what extent these expectations are fulfilled or contradicted.

Especially in the last few years, the study of the Sun (Schou *et al.* 1998, Thompson *et al.* 2003) has broadened our views on the later-type near-main-sequence stars that are involved

in all of LTCBs, NCBs and DCBs. These studies give important knowledge on for example the inner edge of the surface convective zone and on differential rotation. As a result we can apply this information to the sun-like stars. The helioseismic studies of both the convective zone's base and the photosphere indicate the existence of a rotational shear layer. The value of  $\Omega/2\pi$  as a function of depth and latitude is shown in Figure 7 of Thompson *et al.* (2003). This rotational shear is almost certainly the main driver of magnetic activity in the Sun, and presumably is also relevant to other cool stars with surface convection zones. The magnetic activity itself leads to mass loss, and to angular momentum loss, and in rapidly rotating stars such as those considered here these processes can be comparable to, and perhaps more important than, nuclear evolution in driving the evolution. Despite this improvement in understanding, however, the chemical abundances in stars still remain as a problem. We assume here for want of more detailed information that all the stars considered have Solar abundances.

Many papers have been published on the evolution of close binaries, although they have tended to concentrate on more massive (OB) systems. Previous studies on low-mass close binaries have been done by Yungel'son (1971, 1972), Tutukov & Yungel'son (1971), Webbink (1976a,b, 1977a,b,c), Shu, Lubow & Anderson (1976), Shu & Lubow (1981), Vilhu (1982), Hilditch, King & McFarlane (1988), Shaw *et al.* (1996), Nelson & Eggleton (2001), Eggleton & Kiseleva-Eggleton (2002), and references cited by these authors. Hilditch *et al.* (1988) listed 31 C and SD systems, and they studied relations between the physical parameters and the evolution of those systems. They also noted that the primary components of W-type systems seem to be unevolved main sequence stars, while A-type ones are approximately on the TAMS. Shaw *et al.* (1996) presented X-ray observation of NCBs from the ROSAT survey. In this study, they concluded that the evolution of A-type W UMa systems is related to the NCBs. Nelson & Eggleton (2001) have investigated the conservative evolution of close binaries for Case A and divided this case into eight sub-classes (AD, AR, AS, AE, AG, AL, AN, AB) five of which (AD, AR, AS, AE, AG) produce contact binaries.

We have collected from the literature a total of 72 LTCBs (Section 2), and 25 NCBs and 11 DCBs (Section 3), whose parameters we believe are relatively well-determined, and are listed in Tables 1, 2 and 3. The observational relations between these parameters such as M-R, M-T, R-T and M-L are plotted. In Section 4 we discuss evolutionary processes, seeking to link DCBs to LTCBs and NCBs. In Section 5 some evolutionary models are taken into account, and evolutionary chains from DCBs to NCBs and LTCBs are suggested. Finally (Section 6) we propose a new model for heat transport in contact binaries.

## 2. Low-Temperature Contact Binaries (LTCBs)

The components of contact binaries are in physical contact, and hence the interactions of the components are more efficient than those of detached binaries. In particular, it is a remarkable feature of LTCBs that the temperatures of the two components are closely equal (typically to  $\lesssim 5\%$ ), despite the fact that the masses are often as different as a factor of five or more. This is taken to mean that there is a considerable amount of heat transfer between the two components, which can be perceived as confirmation that the two components share a common envelope.

There are two kinds of contact binaries: one group, known as W UMa systems, are extensively studied low-temperature contact binaries (LTCBs) whose components share a common convective envelope, while another group consists of high-temperature contact binaries (HTCBs) with common radiative envelopes. LTCBs have been divided into two subgroups of A- and W-type by Binnendijk (1970). If the more massive component is eclipsed during the deeper (primary) minimum, and is therefore the hotter component, we are dealing with an A-type system, and if the less massive star is the hotter one, the system is called W-type. The spectral types of A-type and W-type systems range from A to G and F to K, respectively.

Various models for contact binaries have been developed: a contact discontinuity (DSC) model (Shu *et al.* 1976, 1979, 1980, Lubow & Shu 1977, 1979); a thermal relaxation oscillation (TRO) model (Flannery 1976, Lucy 1976, Robertson & Eggleton 1977); and an angular momentum loss (AML) model (Mochnacki 1981, Rahunen & Vilhu 1982). Recently Kähler (2002a, 2002b, 2004) has studied the structure of contact binaries. Li *et al.* (2004) developed a model which is based on the model of Robertson & Eggleton (1977). Further discussion of models is by Hazlehurst & Refsdal (1978), Lucy & Wilson (1979), Smith *et al.* (1980), Shu (1980), Eggleton (1981, 1996) and Rucinski (1986). LTCBs have been studied kinematically by Aslan *et al.* (1999).

Relations between physical parameters of LTCB-type contact systems have been discussed by numerous authors (Eggleton 1981; Mochnacki 1981; Maceroni, Milano & Russo 1985; Hilditch, King & McFarlane 1988; Sarna & Fedorova 1989; Rovithis-Livaniou *et al.* (1992), Maceroni & van't Veer 1996, and references therein).

In many cases where there exist both spectroscopic and photometric analyses, the  $q$  value found from spectroscopic studies can be quite different from the resulting value of a photometric  $q$  search; in such cases it is probably the photometric mass-ratio value which is wrong. It is important to obtain both photometric and spectroscopic studies of the system in order to determine the orbital and physical parameters of an eclipsing binary system. For

the systems which appear in Table 1 this criterion has been taken into consideration. All the data contained in the Table is obtained from a combination of spectroscopic and photometric data: DDO – David Dunlop Observatory – and DAO – Dominion Astrophysical Observatory – have greatly contributed to the studies of close binaries’ radial velocities.

In Table 1 well-determined LTCBs are listed. Column 2 shows the type according to Binnendijk’s classification; column 3 gives the spectral type mentioned in the literature and column 4 gives the spectral types of the components based on Popper’s (1980) calibration of spectral type against temperature. Columns 5 – 13 give the period, masses, radii, temperatures and luminosities. Column 14 is the overfill factor  $f$  of equation (2) below, and Columns 15 – 16 are  $X_1 \equiv \ln(R_1/R_{L1})$  and  $X_2 \equiv \ln(R_2/R_{L2})$ , where  $R_L$  is the Roche-lobe radius. Luminosities are calculated from the radii and temperatures by  $L_i/L_\odot = (R_i/R_\odot)^2(T_i/5780)^4$ , where suffix  $i$  refers to either component of the system; but see below for a discussion of the radii.

However, the accuracy of the fundamental parameters is determined not just by the goodness of fit of data points to theoretical curves – radial-velocity curves and eclipsing light curves – but also by uncertainties which we loosely describe as ‘systematic’. Some of these are as follows.

Light curves constrain the temperature ratio  $T_2/T_1$  quite strongly but not the individual temperatures. Often  $T_1$  is estimated on the basis of a spectral type, and these types appear to be often quite old, and/or uncertain. We can expect them to be uncertain, because these stars are rotating ten or more times more rapidly than normal spectral standards. Light curves are normally based on the Roche model of close binaries; we reject a few that are based on simpler models. But the Roche model is clearly only an approximation. Many contact binaries show an asymmetry where one maximum is higher than the other (the O’Connell effect), and such asymmetry is not possible in the Roche model. These asymmetries are usually attributed to ‘spots’, which we interpret here in a very general sense: they might be due to large cool star-spots, to hot regions such as faculae, to gas streams and their impact on the companion star, or to some inhomogeneity not yet understood. We would emphasize that since spots are *necessary* for the interpretation of many light curves, the mere fact that several systems do not show an obvious asymmetry does not justify the assumption that they are free of spots; it may be that in some systems the spot or spots happen to be distributed fairly symmetrically. It is frequently noted that the hypothetical distribution of spots is by no means uniquely defined by fitting a model (Roche plus spots) to a light curve. For example, a hot spot on one side of a star and a cool spot on the other side might well produce much the same light curve, but for different values of mass ratio *etc.*

Table 1. Physical Parameters of Well-Determined LTCBs.

Name	B	sp1	sp2	P	T <sub>1</sub>	T <sub>2</sub>	M <sub>1</sub>	M <sub>2</sub>	R <sub>1</sub>	R <sub>2</sub>	L <sub>1</sub>	L <sub>2</sub>	f	X <sub>1</sub>	X <sub>2</sub>	Ref.
QX And	A	F4V	F4+F5	0.4118	6500	6421	1.18	0.24	1.40	0.70	3.12	0.74	0.210	0.022	0.049	M95
AB And	W	G2V	G8+G1	0.3319	5495	5888	1.01	0.49	1.05	0.77	0.91	0.63	0.130	0.024	0.036	H88d, D02
GZ And	W	F8V	G4+F7	0.3050	5810	6200	1.12	0.59	1.01	0.76	1.05	0.77	0.080	0.016	0.023	B04a
OO Aql	A	G5V	G6+G6	0.5068	5700	5680	1.05	0.88	1.40	1.30	1.85	1.57	0.220	0.058	0.064	H01
V417 Aql	W	F9V	F9+F7	0.3703	6030	6256	1.40	0.50	1.31	0.84	2.02	0.96	0.190	0.029	0.050	S97, L99
SS Ari	W	G0V	G2+F8	0.4060	5860	6123	1.31	0.40	1.37	0.82	1.99	0.84	0.160	0.022	0.041	K03a
AH Aur	A	F7V	F7+F8	0.4941	6215	6141	1.68	0.28	1.89	0.91	4.75	1.06	0.670	0.060	0.143	V01
V402 Aur	W	F2V	F2+F1	0.6035	6700	6775	1.64	0.33	1.98	0.96	7.05	1.75	0.030	0.003	0.007	Z04
TY Boo	W	G5V	G4+F7	0.3171	5800	6180	0.93	0.40	1.13	0.83	1.29	0.90	0.870	0.141	0.215	R90a
TZ Boo	A	G2V	G1+G5	0.2976	5890	5754	0.72	0.11	0.97	0.43	1.02	0.18	0.130	0.011	0.029	H88c, A89
XY Boo	A	F0V	A9+F0	0.3705	7200	7102	0.93	0.15	1.21	0.54	3.54	0.66	0.050	0.005	0.011	M83, A84
CK Boo	A:	F7	F7+F6	0.3551	6200	6291	1.42	0.15	1.48	0.59	2.89	0.48	0.050	0.044	0.126	K04a
EF Boo	W	F5V	F6+F5	0.4295	6338	6450	1.61	0.82	1.50	1.13	3.24	1.97	0.280	0.053	0.076	O04
AO Cam	W	G0 *	G7+G1	0.3299	5590	5900	1.12	0.49	1.09	0.76	1.04	0.62	0.120	0.021	0.033	B04a
DN Cam	W	F5V *	F4+F2	0.4983	6530	6700	1.85	0.82	1.76	1.25	5.05	2.84	0.330	0.057	0.088	B04a
TX Cnc	W	F8V	G1+F7	0.3830	5888	6165	0.91	0.50	1.13	0.87	1.37	0.98	0.210	0.042	0.059	H88c
BH Cas	W	K1	K3+K1	0.4059	4790	4980	0.74	0.35	1.11	0.80	0.58	0.35	0.220	0.040	0.060	M99, Z01
V523 Cas	W	K5V *	K4+K3	0.2337	4410	4736	0.75	0.38	0.75	0.56	0.19	0.14	0.130	0.025	0.036	Z04
RR Cen	A	A9 *	F0+F2	0.6060	6920	6760	2.09	0.45	2.24	1.07	10.31	2.14	0.052	0.012	0.007	H88c, K84b
V752 Cen	W	F8V	G0+F7	0.3700	5955	6221	1.30	0.40	1.27	0.75	1.83	0.76	0.090	0.023	0.023	B93
V757 Cen	W	F9V	G1+F9	0.3432	5900	6000	0.88	0.59	1.01	0.85	1.10	0.83	0.140	0.032	0.040	M84,K84a
VW Cep	W	G9V *	K1+K0	0.2783	5010	5250	0.93	0.40	0.93	0.64	0.49	0.28	0.180	0.031	0.049	H88c, K02a
TW Cet	W	G5V *	G8+G6	0.3169	5450	5630	1.06	0.61	1.00	0.78	0.80	0.55	0.030	0.006	0.009	R82??
RW Com	W	K0V	K0+G8	0.2373	5120	5400	0.56	0.20	0.71	0.46	0.31	0.16	0.170	0.026	0.044	M87
RZ Com	W	G0 *	F7+F5	0.3385	6165	6450	1.23	0.55	1.12	0.78	1.62	0.94	-0.001	0.004	0.004	H88c
CC Com	W	K7? *	K6+K5	0.2210	4170	4365	0.79	0.43	0.75	0.58	0.15	0.11	0.240	0.048	0.067	H88c
$\epsilon$ CrA	A	F2V	F0+F3	0.5914	7100	6640	1.72	0.22	2.12	0.88	10.27	1.34	0.300	0.023	0.063	G93
YY CrB	A:	F8V	F8+F8	0.3766	6135	6142	1.43	0.35	1.45	0.82	2.66	0.86	0.630	0.073	0.146	P02b
SX Crv	A	F6V	F6+F7	0.3166	6340	6160	1.25	0.10	1.31	0.44	2.50	0.25	0.270	0.015	0.051	Z04
DK Cyg	A	A8	A8+F2	0.4707	7500	6700	1.74	0.53	1.70	1.02	8.16	1.89	0.300	0.041	0.075	B04a
V401 Cyg	A	F0V *	F2+F3	0.5827	6700	6650	1.68	0.49	1.98	1.19	7.08	2.49	0.460	0.060	0.112	R02, W00
V1073 Cyg	A	F2V	F2+F3	0.7859	6700	6590	1.60	0.51	2.51	1.64	11.37	4.53	0.920	0.123	0.216	A92
V2150 Cyg	A	A6V	A6+A6	0.5919	8000	7920	2.35	1.89	2.02	1.84	14.94	11.91	0.190	0.049	0.056	K03c
RW Dor	W	K1V	K3+K0	0.2855	4780	5200	0.64	0.43	0.80	0.67	0.30	0.30	0.130	0.030	0.038	H92
BV Dra	W	F9V *	F7+F6	0.3501	6245	6345	1.04	0.43	1.11	0.75	1.69	0.82	0.110	0.019	0.030	K86
BW Dra	W	F8V	F9+F7	0.2922	5980	6164	0.92	0.26	0.98	0.56	1.11	0.41	0.140	0.018	0.035	K86
EF Dra	A:	F9V	F9+F8	0.4240	6000	6054	1.81	0.29	1.72	0.80	3.43	0.77	0.460	0.041	0.099	P01b



The analysis of radial-velocity curves generally assumes that they reflect the motions of the centers of gravity of the two components. However the spots which can influence the photometry can also influence the spectroscopy. In an ideal world, the individual spectra would be fitted to synthetic spectra at the same time that light curves in at least three colors are fitted to synthetic light curves. But even in this ideal world it is unlikely that a unique spotted Roche model would emerge.

In preparing this compilation (but more especially Table 2, containing probable SD systems) we have noticed that where a component is described as filling or overfilling its Roche lobe the quoted stellar radius  $R$  (by which is normally meant the volume-radius) can be quite significantly *smaller* than the minimal value expected from the Roche-lobe radius  $R_L$ .  $R_L/a$  is a function of  $q$  only, and is approximated to an accuracy which is never worse than 0.83% by

$$\frac{R_L}{a} = \frac{0.49q^{2/3}}{0.6q^{2/3} + \ln(1 + q^{1/3})} \quad , \quad 0 < q < \infty, \quad (1)$$

(Eggleton 1983). In the context of Table 1 (contact binaries), we have found several cases which seemed worthy of inclusion, but where  $\ln(R_1/R_{L1})$  was *less* than zero, by as much as 0.08 (indicating underfill by  $\sim 8\%$ ). We used the  $R_1$ ,  $q$ , period and  $M_1$  as given in the literature, as well as Eq. (1), to determine  $\ln(R_1/R_{L1})$ . Probably the reason for the discrepancy is that for such highly non-spherical components the term ‘radius’ is ambiguous. However the volume-radius is unambiguous, except to the extent that in contact systems we somewhat arbitrarily cut the volume into two unequal pieces by a plane through the L1 point perpendicular to the line-of-centres. In order to obtain consistency we have adjusted the radii to correspond (approximately) to the volume radii, according to the following prescription.

Most photometric analyses of contact binaries express the degree of overfill by a factor  $f$  (Column 14 of Table 1) which is

$$f \equiv \frac{\Omega - \Omega_L}{\Omega_{OL} - \Omega_L} \quad . \quad (2)$$

Here  $\Omega$  is the (dimensionless) potential on the joint surface of the star, and  $\Omega_L$ ,  $\Omega_{OL}$  are the potentials of the inner and outer critical lobes, respectively. For modest degrees of overfill (and also underfill), we expect that  $f \approx C(q) \ln(R_1/R_{L1})$ , purely from Roche geometry. We estimate  $C(q)$  by using the following approximation for  $R_{OL}/a$ , the fractional outer-lobe volume radius, which in the same spirit as Equn (1) is

$$\frac{R_{OL}}{a} \approx \frac{0.49q^{2/3} + 0.15}{0.6q^{2/3} + \ln(1 + q^{1/3})} \quad , \quad q \geq 1 \quad , \quad (3)$$

$$\approx \frac{0.49q^{2/3} + 0.27q - 0.12q^{4/3}}{0.6q^{2/3} + \ln(1 + q^{1/3})} \quad , \quad q \leq 1 \quad . \quad (4)$$

Table 1—Continued

Name	B	sp1	sp2	P	T <sub>1</sub>	T <sub>2</sub>	M <sub>1</sub>	M <sub>2</sub>	R <sub>1</sub>	R <sub>2</sub>	L <sub>1</sub>	L <sub>2</sub>	f	X <sub>1</sub>	X <sub>2</sub>	Ref.
FU Dra	W	F8V	G4+F8	0.3067	5800	6133	1.17	0.29	1.13	0.62	1.29	0.48	0.240	0.029	0.058	V01
YY Eri	W	G5V *	G9+G7	0.3210	5362	5600	1.54	0.62	1.20	0.80	1.06	0.56	0.100	0.017	0.027	N86, Y99, M94
QW Gem	W	F8V	G1+F8	0.3581	5890	6100	1.31	0.44	1.26	0.79	1.72	0.77	0.230	0.033	0.059	K03c
V728 Her	W	F3V	F3+F1	0.4713	6622	6787	1.65	0.30	1.81	0.92	5.65	1.59	0.710	0.067	0.153	N95
V829 Her	W:	G2V	G1+G9	0.3581	5900	5380	0.86	0.37	1.07	0.74	1.25	0.42	0.200	0.034	0.054	Z04
V842 Her	W	F9V	F9+F6	0.4190	6000	6280	1.36	0.35	1.46	0.81	2.47	0.92	0.250	0.030	0.061	N96, R99
EZ Hya	W	F9V	G6+F8	0.4497	5721	6100	1.37	0.35	1.55	0.87	2.30	0.94	0.340	0.041	0.082	Y04c
FG Hya	A:	G2V	G1+F9	0.3278	5900	6012	1.44	0.16	1.42	0.59	2.20	0.41	0.860	0.059	0.165	Q05, L99
SW Lac	W	F8V **	G9+G6	0.3207	5347	5630	0.98	0.78	1.03	0.94	0.78	0.80	0.310	0.078	0.089	B04
XY Leo A	W	K2V	K4+K2	0.2841	4524	4850	0.82	0.50	0.86	0.69	0.28	0.23	0.060	0.013	0.017	Y03a
AP Leo	A:	F7	F8+F7	0.4304	6150	6250	1.46	0.43	1.46	0.85	2.75	0.98	0.060	0.008	0.015	K03c
VZ Lib	A	G0V *	G1+G9	0.3583	5900	5380	1.48	0.38	1.33	0.73	1.92	0.40	0.130	0.016	0.032	Z04
UV Lyn	W	F6V *	F9+F7	0.4150	6045	6262	1.36	0.50	1.45	0.96	2.52	1.28	0.460	0.070	0.117	V01
TV Mus	A:	F8V	F9+F8	0.4457	5980	6088	0.94	0.13	1.41	0.59	2.27	0.43	0.130	0.011	0.028	H89
V502 Oph	W	G2	G1+F7	0.4534	5880	6165	1.38	0.48	1.45	0.89	2.25	1.03		0.000	-0.008	H88c
V508 Oph	A	F9V	F9+G3	0.3448	6000	5830	1.01	0.52	1.07	0.80	1.33	0.66	0.100	0.019	0.028	L90
V566 Oph	A	F1V	F0+F1	0.4096	7000	6881	1.40	0.33	1.47	0.79	4.65	1.25		0.035	0.068	N03
V839 Oph	A	F7V *	F3+F4	0.4090	6650	6554	1.64	0.50	1.50	0.90	3.94	1.33	0.230	0.031	0.058	P02a
V2388 Oph	A	F3V	F1+F6	0.8023	6900	6349	1.80	0.34	2.64	1.35	14.12	2.63	0.650	0.063	0.142	Y04b
ER Ori	W	F8V *	F7+F6	0.4234	6200	6314	1.53	0.98	1.40	1.15	2.60	1.90	0.150	0.034	0.043	G94
U Peg	W:	G2V	G2+G3	0.3748	5860	5841	1.15	0.38	1.25	0.78	1.65	0.63	0.240	0.035	0.061	P02b
AE Phe	W	G1	G1+F7	0.3624	5888	6166	1.38	0.63	1.26	0.90	1.72	1.05	0.210	0.037	0.057	H88c, B04b
BX Peg	W	F8V **	G9+G7	0.2804	5300	5528	1.02	0.38	0.97	0.63	0.67	0.34	0.190	0.030	0.050	S91
OU Ser	W	G0V	G0+F6	0.2968	5960	6380	1.02	0.18	1.09	0.52	1.36	0.40	0.310	0.029	0.069	P02b
Y Sex	A	F8	F7+F8	0.4198	6210	6093	1.21	0.22	1.50	0.75	3.01	0.70	0.640	0.061	0.139	Y03b
RZ Tau	A	A8V *	A9+A9	0.4157	7300	7194	1.70	0.64	1.58	1.07	6.38	2.76	0.550	0.084	0.139	Y03c
EQ Tau	A	G2V	G2+G2	0.3413	5860	5851	1.22	0.54	1.15	0.80	1.39	0.68	0.120	0.021	0.033	P02b
V781 Tau	W	G0V	G7+G0	0.3449	5621	5940	1.24	0.55	1.15	0.80	1.18	0.71	0.060	0.011	0.017	Y04a
AQ Tuc	A	A9V *	F0+F1	0.5948	6980	6860	1.93	0.69	2.05	1.33	8.93	3.53	0.370	0.055	0.094	H86b, C01
W UMa	W	F8	F9+F6	0.3340	6026	6310	1.35	0.69	1.15	0.85	1.56	1.03	0.080	0.016	0.022	H88c, R93
AA UMa	W	F9V	G0+F9	0.4681	5929	5965	1.56	0.85	1.47	1.11	2.39	1.40		-0.006	-0.010	B93
AW UMa	A	F0	A9+F0	0.4387	7175	7022	1.79	0.14	1.90	0.68	8.60	1.01	0.850	0.046	0.150	P99
HV UMa	A	A2V **	F0+F0	0.7108	7000	7000	2.80	0.50	2.86	1.44	17.60	4.48	0.770	0.072	0.165	C00
AH Vir	W	G2V **	G9+G6	0.4075	5300	5671	1.36	0.41	1.41	0.84	1.40	0.65	0.230	0.031	0.058	L93
GR Vir	A	F7/8V	F6+F7	0.3470	6300	6163	1.37	0.17	1.43	0.62	2.90	0.50	0.790	0.058	0.156	Q04

References. — References are identified by the initial of the first author, the last two digits of the date, and letters  $a, b, \dots$  in case of ambiguity. The same abbreviation is given in the reference list at the end of this paper.  
 $P$  in days, masses, radii and luminosities in solar units, temperatures in Kelvins.  $x \equiv \ln(R/R_L)$

The discontinuity of gradient at  $q = 1$  is real, not an artefact of the approximation. The accuracy is better than about 2%, over all  $q$ . Assuming that the Roche potential varies approximately linearly with the log of the volume radius in the rather narrow range involved, we estimate  $R$  from

$$\ln R \approx \ln R_L + (\ln R_{OL} - \ln R_L)f \quad . \quad (5)$$

We have used this equation to adjust the radii (normally by less than  $\sim 3\%$ ) in all cases except six where  $f$  was not cited.

Another discrepancy that we have sometimes noted is between the spectral type and the temperature. We mentioned above that the spectral type can be quite uncertain, because of the rapid rotation. However, photometric analysis often starts by noting that because the spectrum has been determined elsewhere to be of a certain type, the temperature of the hotter star is assumed to have a value consistent with some specific calibration of temperature versus spectral type. However, it can be seen in Table 1 that the spectrum (‘sp1’) listed in Column 3 and taken from the literature cited is often not very consistent with the temperature adopted in Column 6 – or Column 7 if the secondary is hotter. Column 4 (‘sp2’) gives our estimate, based on the calibration of Popper (1980), of the spectral types of the two components, *assuming* that the temperatures in Columns 6 and 7 are correct. It can be seen that ‘sp1’ quite often falls outside the range of ‘sp2’, and it is not clear therefore whether either of  $T_1$  or  $T_2$  is correct.

We might note that if the system is W-type, then we would expect the spectral type to determine mainly the *cooler* temperature, since the cooler component dominates the luminosity. This is only a small effect since the temperatures are always fairly similar. But the discrepancy noted in the previous paragraph seems to imply a substantial uncertainty,  $\sim 10\%$ , in the temperatures of *some* systems. There is probably much less uncertainty in the *ratio* of temperatures, since this is usually rather well determined by the ratio of the depths of primary and secondary eclipse.

A very minor uncertainty that we note is that in some cases the Binnendijk type (Column 2) taken from the literature is at odds with the ratio of temperatures, also taken from the literature. We have noted such cases with a colon. Since in many cases the difference in temperature is so slight as to be insignificant, it might be a good idea to introduce a third Binnendijk type: N for Neither.

In Figure 1, the relations of  $M-R$ ,  $M-T$ ,  $R-T$  and  $M-L$  of LTCBs are shown. In this figure the squares represent W-type LTCBs and plusses represent A-type LTCBs; the red symbols indicate the primary components and the green ones the secondary components. Figure 2 shows the HR diagram for the primaries and secondaries of LTCBs, NCBs and DCBs. In Figure 2 filled circles show DCBs and open circles indicate NCBs.

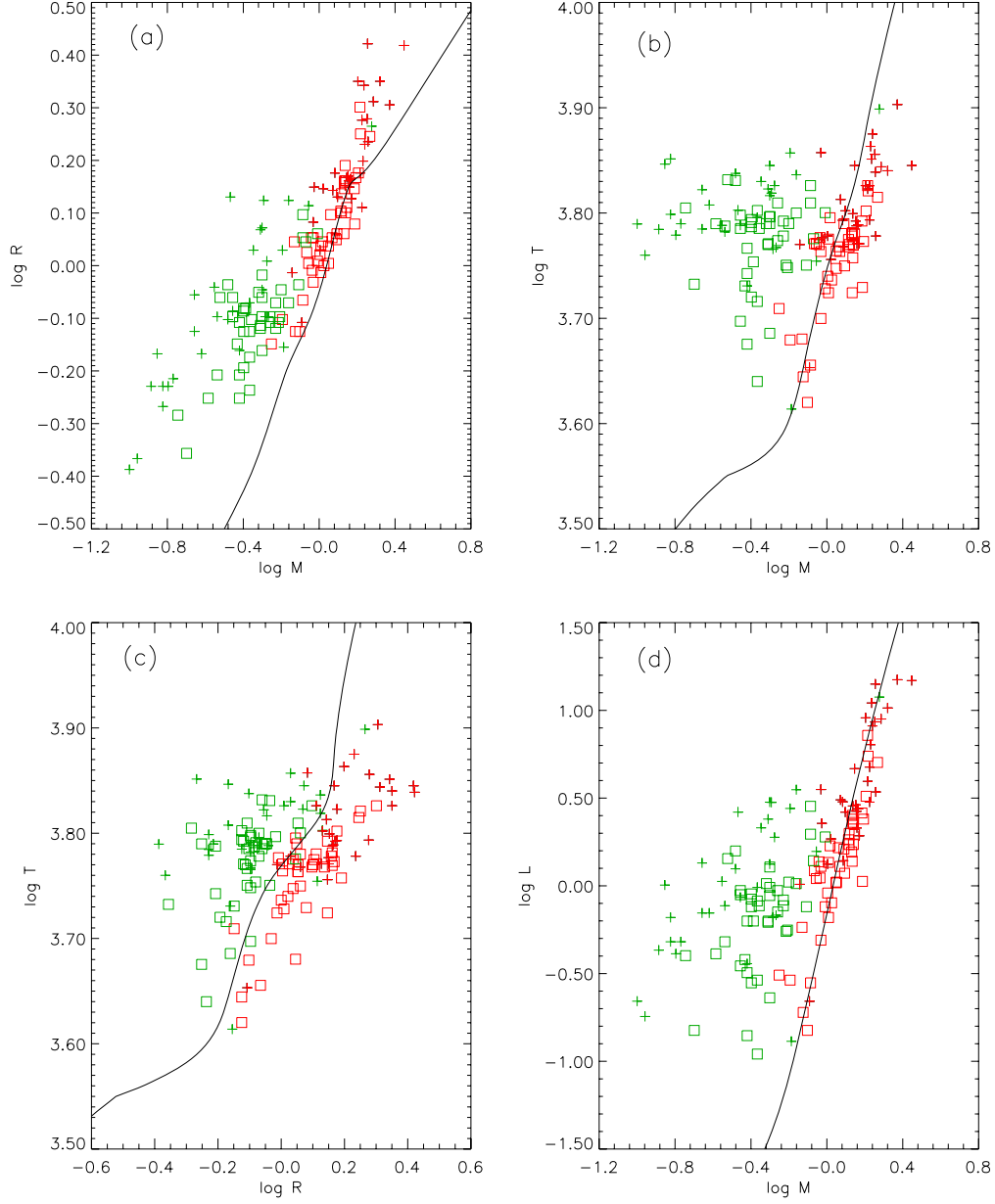


Fig. 1.— The M-R, M-T, R-T and M-L planes of LTCBs. The squares represent the W-type LTCBs, and pluses the A-type LTCBs; red indicates the primary component and green the secondary component.

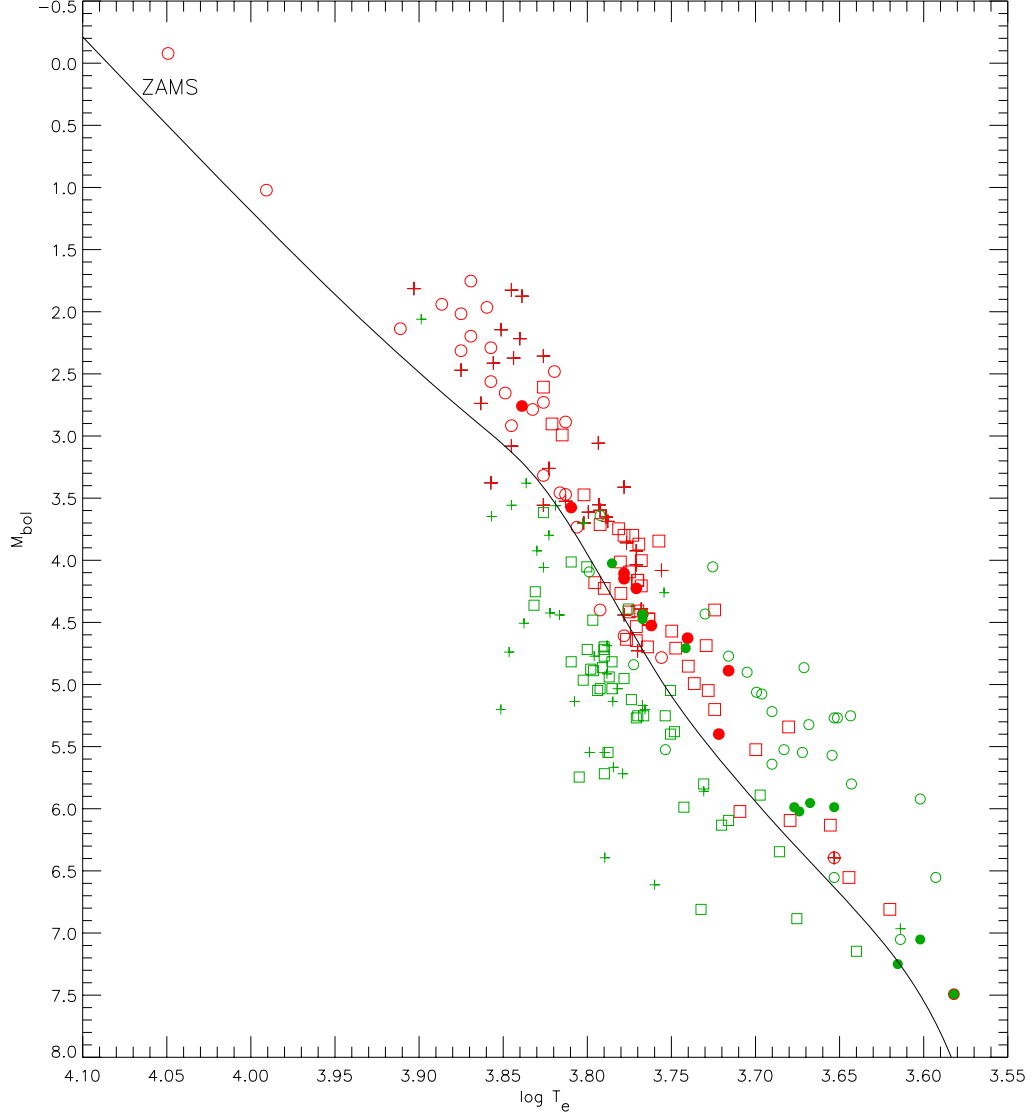


Fig. 2.— The HR diagram for LTCBs, NCBs and DCBs. The ZAMS line is taken from Pols et al. (1995). The W-type LTCBs, A-type LTCBs, NCBs and DCBs are shown by squares, plusses, open circles and filled circles, respectively. The primaries of each type are red, the secondaries green.

### 3. Near-Contact Binaries (NCBs) and Detached Close Binaries (DCBs)

Table 2 is a selection of 25 close binaries ( $P < 1$  d) where both components are very close to their Roche lobes, but where in contrast to the LTCBs the temperature difference is quite large. In view of the fact that LTCBs apparently manage to maintain rather closely equal temperatures, presumably through transport of energy from the more massive to the less massive component in a shared envelope, it seems likely that NCBs are either (a) in such shallow contact that energy transport cannot be effective, (b) they are semidetached, or (c) they are detached. We shall argue that the evidence supports (b), although we cannot rule out that one or two may be (a), and one or two others (c). If some are indeed (a), it seems very reasonable to assume that their depths of contact must be smaller than in any LTCBs. Table 3 lists a selection of 11 detached close binaries (DCBs), also with  $P < 1$  d, where both stars are clearly well within their Roche lobes, and not coincidentally both components appear to be reasonably normal lower MS stars.

Table 2. Some Near-Contact Binaries (NCBs)

name	type	$P$	spect.	$M_1$	$M_2$	$R_1$	$R_2$	$T_1$	$T_2$	$L_1$	$L_2$	$X_1$	$X_2$	$t_p$	refs
BX And	C	0.6101	F1	1.52	0.75	1.79	1.30	6800	4500	6.10	0.62	0.016	0.021	-11.0	B90b
CN And	SD1	0.4628	F5	1.30	0.50	1.42	0.92	6500	5922	3.25	0.92	0.005	-0.007	-2.2	vH01,R00b
CX Aqr	SD2	0.5560	F5	1.20	0.66	1.30	1.16	6400	4969	2.55	0.74	-0.150	0.012		H86a
EE Aqr	D	0.5090	A9.5	2.20	0.71	1.76	1.07	7060	4395	6.89	0.38	-0.043	-0.026		C90,H88b
DO Cas	C	0.6847	A7	1.70	0.53	2.06	1.21	7700	4000	13.3	0.34	0.000	0.000		O92,K85,M58
YY Cet	SD2	0.7905	A8	1.84	0.94	2.09	1.63	7500	5314	12.4	1.90	-0.059	-0.001		M86b
W Crv	SD2	0.3881	G6	1.00	0.68	1.01	0.92	5700	4900	0.97	0.44	-0.082	-0.004	34.0	R00a,O96
RV Crv	SD2	0.7473	F3	1.64	0.45	2.18	1.21	6600	5070	8.08	0.87	-0.004	0.001		M86a
GO Cyg	SD1	0.7178	B8	3.08	1.63	2.46	1.45	11200	6200	85.4	2.78	0.000	-0.240	6.2	R90b,O54,S85,P33
V836 Cyg	SD2	0.6534	B9.5	2.20	0.78	1.94	1.34	9790	5370	31.0	1.34	-0.101	0.000	8.0	Y04a,D82
RZ Dra	SD2	0.5500	A5	1.63	0.65	1.67	1.12	8150	4900	11.1	0.65	-0.023	-0.001		R00b,N03
BL Eri	SD	0.4169	F9	0.61	0.33	0.99	0.73	6000	5670	1.14	0.49	-0.007	-0.032		Y88
TT Her	SD1	0.9121	A9	1.56	0.68	2.30	1.44	7239	4690	13.0	0.90	-0.018	-0.108	-5.0	M89,S37
RS Ind	D	0.6241	A9	2.00	0.62	2.00	1.18	7200	4659	9.64	0.59	-0.022	-0.016	-3.0	H88b
FT Lup	SD1	0.4701	F2	1.39	0.65	1.44	0.96	6700	3916	3.74	0.19	-0.001	-0.059	-2.7	L86,H84
SW Lyn	SD1	0.6441	F2	1.72	0.90	1.89	1.31	6700	4480	6.43	0.62	0.000	-0.069		K03c,L01
V361 Lyr	SD1	0.3096	F7	1.26	0.87	1.02	0.72	6200	4500	1.38	0.19	0.003	-0.185	-11.0	H97
V1010 Oph	SD1	0.6614	A8	1.41	0.68	1.82	1.22	7500	5200	9.43	0.98	0.008	-0.057	-1.6	S90,C91,G77,L87
DI Peg	SD2	0.7118	F4	1.19	0.70	1.41	1.37	6550	4400	3.29	0.63	-0.223	-0.008		L92
VZ Psc	SD1	0.2613	K4	0.82	0.65	0.78	0.68	4500	4110	0.22	0.12	0.000	-0.028		H95
RT Scl	D	0.5116	F0	1.63	0.71	1.59	1.01	7000	4820	5.41	0.49	-0.020	-0.092	-4.0	H86b,D79
RU UMi	SD2	0.5249	A9	2.51	0.78	1.76	1.16	7200	5005	7.50	0.75	-0.108	-0.001		M96
AG Vir	D	0.6427	A8	1.67	0.53	1.97	1.14	7400	6293	10.5	1.83	0.006	-0.016		B90a
CX Vir	SD2	0.7461	F5	1.06	0.36	1.86	1.13	6500	4513	5.56	0.47	0.008	-0.005		H88a
FO Vir	SD2	0.7756	A8	1.75	0.27	2.42	1.05	7400	4700	15.8	0.48	-0.004	0.000		S91,M86c

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$P$  in days, masses, radii and luminosities in solar units, temperatures in Kelvins.

$X \equiv \ln(R/R_L)$ ;  $t_p$  is the observed timescale of period change (Myr), but alternative interpretations are possible.

NCBs are more difficult targets than LTCBs, because the ratio of luminosities is more

extreme. Even though *both* components are very close to filling their lobes, the substantially lower temperature of the secondary makes its luminosity typically less than a half of what it would be if at the same temperature as the primary. Whereas the luminosity ratio in an LTCB is typically 4 or 5, in an NCB it is usually greater than 10. Thus measurably double-lined systems will be harder to come by. Nevertheless 22 of the 25 NCBs in Table 2 are double-lined; in fact we largely exclude single-lined systems, on the grounds that their fundamental parameters are not well-enough determined. As with LTCBs, mass ratios can in principle be determined from light-curve analysis independently of radial-velocity data, but such photometric determinations tend to have considerably greater uncertainty than spectroscopic determinations. We do however admit 3 SB1 systems, DO Cas, TT Her and V836 Cyg, believing that these are relatively secure.

There are several determinations of the fundamental parameters of NCBs, not included in Table 2, where in the absence of sufficient spectroscopic data the assumption has been made that the primary has a ‘normal’ mass for its spectral type. This may be a reasonable assumption; indeed our Fig 3b below largely but not entirely supports it. However, it is obvious that we will not be able to assess the reliability of this assumption if we include several systems for which it has already been made.

In Table 2, Col. 2 lists the geometrical configuration suggested by at least one of the references cited: SD1 means semidetached, with star 1, the more massive, filling its Roche lobe, SD2 means that star 2 fills its Roche lobe, C means that both components fill their lobes and D that neither does. Col. 3 is the period (days), and Col. 4 is the spectral type, as inferred from the temperature of the primary and the compilation of Popper (1980). Cols 5 – 12 give the masses, radii, temperatures and luminosities of the two components, in solar units or Kelvins. Cols 13 and 14 are  $X_1 \equiv \ln(R_1/R_{L1})$  and  $X_2 \equiv \ln(R_2/R_{L2})$ , measuring the underfill or overfill of the two components. Col. 15 is an estimate of the timescale (Myr) of period change, from long-term monitoring of times of eclipses. It should be noted that such estimates are very uncertain, since the observed period changes can often be attributed alternatively to the presence of a third body, and/or cyclic magnetic activity. Table 3 has the same format, except that Col. 2 is missing since they are all of type D.

Table 3. Some Detached Close Binaries (DCBs)

name	$P$ (d)	spect.	$M_1$	$M_2$	$R_1$	$R_2$	$T_1$	$T_2$	$L_1$	$L_2$	$X_1$	$X_2$	$t_P$	refs
RT And	0.6289	G0 + K3	1.13	0.83	1.22	0.89	5900	4651	1.62	0.33	-0.250	-0.427	-10.0	P00
SV Cam	0.5931	F5 + K4	1.46	0.87	1.38	0.94	6450	4500	2.95	0.32	-0.193	-0.339		K02b
WY Cnc	0.8294	G5 + K7	1.17	0.60	1.17	0.72	5500	4000	1.12	0.12	-0.521	-0.702		P98,K04c
VZ CVn	0.8425	F0 + F8	1.84	1.43	1.76	1.25	6900	6100	6.26	1.95	-0.242	-0.463		P88,C77
CG Cyg	0.6311	G9 + K3	0.94	0.82	0.89	0.84	5270	4720	0.55	0.31	-0.498	-0.490		P94,Z94
YY Gem	0.8143	M1 + M1	0.61	0.61	0.63	0.63	3820	3820	0.08	0.08	-0.856	-0.859		S00,P80
UV Leo	0.6001	G0 + G2	1.13	1.08	1.13	1.13	5848	5848	1.34	1.34	-0.279	-0.260		P97,F96
UV Psc	0.8610	G5 + K3	0.98	0.77	1.11	0.83	5780	4753	1.23	0.32	-0.506	-0.682		P97,B96
XY UMa	0.4790	G9 + K6	1.10	0.66	1.16	0.63	5200	4125	0.88	0.10	-0.128	-0.506		P01a
BH Vir	0.8169	F8 + G5	1.18	1.05	1.23	1.11	6000	5850	1.74	1.29	-0.423	-0.469		K04b
ER Vul	0.6981	G0 + G8	1.16	1.05	1.25	1.12	6000	5514	1.81	1.04	-0.292	-0.357		K03b

References are identified by the initial of the first author, the last two digits of the date, and letters  $a, b, \dots$  in case of ambiguity. The same abbreviation is given in the reference list at the end of this paper.

$P$  in days, masses, radii and luminosities in solar units, temperatures in Kelvins.

$X \equiv \ln(R/R_L)$ ;  $t_P$  is the observed timescale of period change (Myr), but alternative interpretations are possible.

NCBs and DCBs, like LTCBs, are subject to ‘spottedness’, in the general sense of Section 2. We can hope that it may be possible to transform a two-dimensional map of intensity as a function of wavelength and time, with suitably high resolution in both dimensions, into a two-dimensional map of temperature as a function of latitude and longitude. But such a transformation is by no means guaranteed to be unique; and usually the data set consists of (a) light curves in a few colours, and (b) radial velocity curves, in which any fine detail due to spottedness is integrated out. Hilditch *et al.* (1997) found that the highly distorted light curve of V361 Lyr could be represented by 4 cool spots on the primary and 2 hot spots on the secondary in 1988, and 3 cool spots and 1 hot spot in 1989. Hrivnak *et al.* (1995) found that the relatively symmetric light curve of VZ Psc required two symmetrically-positioned hot spots on the secondary.

The assumption is normally made that there exists an unchanging set of fundamental parameters, e.g. the masses and the Roche potentials (or equivalently the volume radii) of the two surfaces, with time-varying perturbations (spots) added to them. It is certainly reasonable to assume that the masses do not change perceptibly on a timescale of decades or centuries. Even though components may be losing and/or exchanging mass, the timing of eclipses shows that this is on a timescale of Myrs at least. But we cannot suppose that the volume-radii are as constant. A star is unlikely to change its *overall* structure, including its radius, on less than a thermal timescale of  $\sim$  Myrs, but the outer 10% by radius, say, could change its structure much more rapidly since its mass and thermal energy are much smaller than overall. Changes might be due to significant magnetic field and its variation, in the sub-photospheric region. As in the case of Table 1, we have amended some radii to ensure that components that are cited as filling their lobes do in fact do so.



Thus it is not clear that meaningful, i.e. constant, radii exist, let alone are accurately measurable. Since the brightness of these systems *outside* eclipse can change by 20% or more on a timescale of years, it is not impossible that the radii, and/or mean temperatures, also change by a few percent. The most optimistic assessment is that the radii and the mean temperatures do not change as much as that, and that the varying light level as seen from Earth is because of time-varying anisotropy of the spotted temperature distribution.

We include the DCBs of Table 3 in our discussion because (a) since both components are fairly far inside their Roche lobes it is reasonable to assume in the first instance that they are normal MS stars, and (b) nevertheless they all have difficult light curves, often with asymmetries and with variability (apart from eclipses) on timescales from days to decades, as do the NCBs and LTCBs. But although they show somewhat similar spottedness to the NCBs and LTCBs, we can reasonably exclude RLOF as a mechanism for this, and therefore learn more about specifically magnetic spottedness.

To what extent are the primaries and secondaries of NCBs and DCBs normal for MS stars? It is quite clear that the secondaries of NCBs are very abnormal, being usually much larger and hotter than MS stars of the same masses. Obviously we should attempt to attribute this to an evolutionary history involving RLOF, whether current or previous. But the other three categories are not so obviously anomalous, and indeed we might hope that both of the components of the DCBs are normal. Ideally we would plot the components in 3-D  $M, R, T$  space, where they should all lie on a surface if they are normal MS stars (of Solar composition) – and on a line if of too low mass to evolve significantly. This being difficult, we compromise with three projections of this space, in Figs 2a – 2c. Anomalies are likely to show up in at least one of these projections.

Fig 3a shows both components of both the NCBs and the DCBs in the  $(\log M, \log R)$  plane. The continuous line is the ZAMS from Pols *et al.* (1995), and the dotted line is the same displaced by a factor of 2 upwards. It can be seen that both components of the DCBs agree reasonably well with the ZAMS. Two components are slightly below it – the primary of SV Cam, and the secondary of VZ CVn – and most of the remaining components are about 10–15% above it. The discrepancy of SV Cam is well within the limits of uncertainty ( $\sim 5\%$ ) given by Kjurkchieva *et al.* (2002). The radii for VZ CVn rely on a light-curve analysis (Cester *et al.* 1977) that is rather primitive by modern standards (the WINK method). The light curve has significant distortion and considerable scatter, and the system would repay a study with modern detectors and analysis. However, as Popper (1988) discussed at length, the main anomaly in VZ CVn is in the temperatures of both components (Fig 3b).

The fact that almost all of the other components of DCBs are somewhat above the ZAMS, by about the same amount, makes their discrepancy appear arguably significant, even

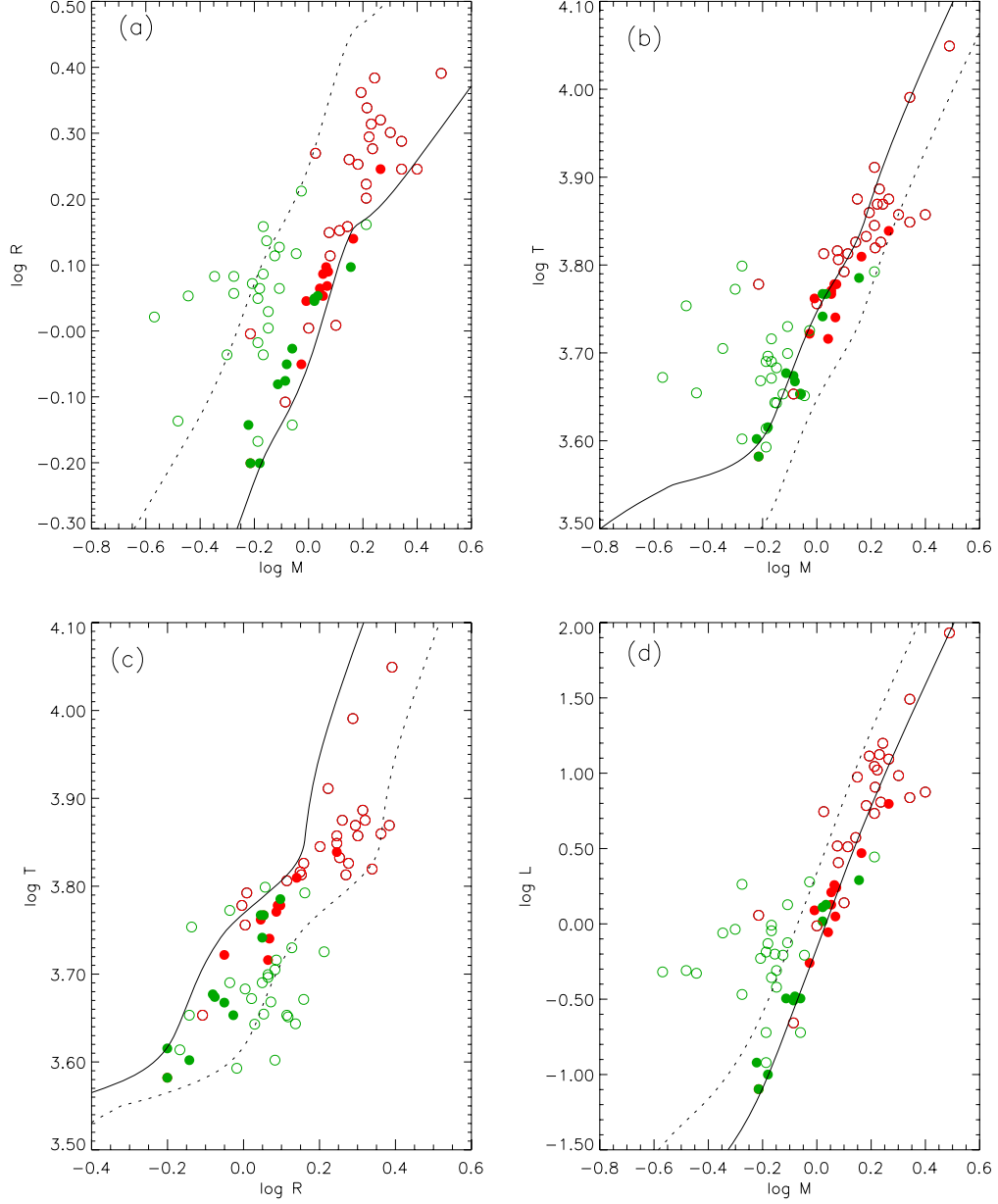


Fig. 3.— Plots of (a)  $R$  vs.  $M$ , (b)  $T$  vs.  $M$ , (c)  $T$  vs.  $R$  and (d)  $L$  vs.  $M$ . NCBs are open circles, DCBs are filled circles; primaries are red and secondaries are green. The ZAMS is the continuous line in (a) – (d). The dotted line is the same line displaced up by 0.3 (a), down by 0.1 (b), right by 0.2 (c), and up by 0.5 (d).

although in any one case we might appeal to measurement uncertainty. It is possible that the discrepancy relates to the fact that they are all active RS CVn systems, in which dynamo activity, exaggerated by rapid rotation, plays a major role. It may be that they are inflated somewhat by an accumulation of magnetic field in their outer layers. All the systems have asymmetric and time-varying distortions that are usually attributed to massive starspots or starspot clusters. There is however at least one alternative possibility: that significant mass loss on a timescale comparable to the nuclear timescale of a  $\sim 1 M_{\odot}$  star ( $\sim 10$  Gyr), much enhanced by very rapid rotation, has affected by 10 – 20% a star whose *initial* mass was over slightly over  $1 M_{\odot}$ , and so was able to undergo some evolutionary expansion before its mass was reduced to a value too low for nuclear evolution: see Section 5.

The Terminal Main Sequence (TMS) is not well defined for stars with mass  $\lesssim 1.5 M_{\odot}$ , and is in any case irrelevant for stars below  $\sim 1 M_{\odot}$  which are not expected to evolve significantly. But the fact that almost all primaries of NCB systems have radii between 1 and 2 times the ZAMS value, and (with one exception: BL Eri, see below) much nearer the ZAMS value for  $M_1 \lesssim 1.1 M_{\odot}$ , is consistent with their being in a long-lived phase of evolution. Only the primary of V361 Lyr (and also the secondary) is arguably significantly *below* its ZAMS radius. The discrepancy is quite significant according to the error estimates of Hilditch *et al.* (1997). However the system has probably the most distorted light curve of any NCB, with the first maximum (after primary eclipse) being brighter than the second by 0.3 mag. Light curves from different seasons gave different geometrical solutions, so there must be considerable uncertainty of a systematic character in the solution. It is however by no means unreasonable that a ZAMS star losing mass on a thermal timescale, as would be expected in an SD1 state, should be undersized for its mass.

One primary (BL Eri) is an outlier in the opposite direction. This primary is very anomalous, with a much lower mass than any other. However the two radial-velocity curves (Yamasaki *et al.* 1988) are defined by only 7 and 6 points, with considerable scatter. We feel that it is preliminary to take this as a firmly established anomaly. Yamasaki *et al.* (1988) point to the need for more data points at higher S/N.

Almost all the secondaries of the NCBs lie *well* above the ZAMS, in Fig 3a. Only two lie below, GO Cyg and V361 Lyr. We have mentioned that the latter system has an unusually distorted light curve, and may therefore have unusually uncertain parameters. However it is also one of the most convincing cases of RLOF, which may well modify its fundamental parameters. The former system relies on the radial-velocity determination of Pearce (1933), as modified by Ovenden (1954) to allow for the unusually strong reflection effect. There is some substantial uncertainty here, which a modern analysis of radial velocity and light curves simultaneously might reduce.

In Fig 3b we see that as in Fig 3a the DCBs are all fairly close to the ZAMS, the *upper* curve, except for the two components of VZ CVn. There is slightly more scatter, which may mean that temperatures are somewhat less secure than radii, or that magnetic effects cause more fluctuation in temperature than in radius. The two components of VZ CVn appear as very evolved in this diagram, despite their *low* radii in Fig 3a. Two NCB primaries are well to the left of the ZAMS: BL Eri (again) and CX Vir. Hilditch & King (1988) estimate the uncertainty of the primary mass in CX Vir as about 15% (largely because  $K_2$  has considerable scatter), and this would permit the primary to have a normal ZAMS mass for its temperature. It would still have too large a radius for its mass, however, to be on the ZAMS, and might be affected by RLOF. Four other NCB primaries that are slightly above the ZAMS line are probably not significantly above it.

Two NCB primaries are conspicuously far to the right of the ZAMS line in Fig 1b: RU UMi and EE Aqr. Both appear to be rather well-determined systems, with RU UMi being the more anomalous. In terms of  $T$  *vs.*  $M$  RU UMi is well beyond the TMS, but in terms of  $R$  *vs.*  $M$  it is only halfway across the MS band. We may be seeing the effect of thermal-timescale RLOF, although we would then expect a period decrease on a timescale of less than a Myr.

The NCB secondaries are almost all hotter, usually substantially, than ZAMS stars of the same mass. Four, GO Cyg, SW Lyn, FT Lup and V361 Lyr, are somewhat cool. However, because the secondaries are so different from ZAMS stars, presumably because of either previous nuclear evolution or thermal-timescale RLOF or both, it is not clear that these four are any more anomalous than the others.

Fig 3c shows again that most components of the DCBs are not far from the ZAMS (the upper curve in this projection). However the deviations of a few are rather more conspicuous here: some are rather too cool *and* rather too large for their masses, including YY Gem which seems rather normal in Figs 2a and 2b. We still suppose that this may be because of their dynamo activity. The primaries of NCBs fall roughly within the expected area. The fact that BL Eri is not anomalous in this plot leads us to suppose that its masses need redetermination at higher S/N (Yamasaki *et al.* 1988).

Fig 3d shows  $\log L$  *vs.*  $\log M$ . The DCBs cluster closer to the ZAMS line than in the previous three plots, suggesting that indeed lower  $T$  goes with higher  $R$ , e.g. that although perhaps magnetic field causes expansion of the envelope the overall luminosity is unchanged. The three highest masses, both components of VZ CVn and the primary of SV Cam, are all somewhat below the ZAMS.

We feel that BL Eri and GO Cyg need substantially improved spectroscopy, and that

VZ CVn needs a more modern photometric analysis; but the latter system has very well-determined masses, and the temperatures are certainly anomalous even if the radii are re-determined to be more normal. Among the remaining systems accuracies may well be of order 10 – 15%, and better in the case of most DCBs. Magnetic activity may be the cause of mildly greater radii and lower temperatures than expected in both components of DCBs, but for those with masses over say  $0.8 M_{\odot}$  – most of them – it could be that rotation-enhanced mass loss has allowed some of them to have evolved appreciably when they were more massive (Section 5). Primaries of NCBs in SD1 systems can be expected to have lower temperatures and radii for their mass and age, because of RLOF, although in SD2 systems we would expect anomalies to be smaller and in the opposite direction.

On theoretical grounds it is likely that most or all of the NCBs should be semidetached (SD1 or SD2), and *not* D or C. If D, there seems no good reason why both components should not be regular MS stars – although once again rotationally-enhanced mass loss can affect this; and if C, why the temperatures are not nearly equal as in LTCBs. However the actual determination of whether a particular system is SD1 or SD2 is by no means easy. The observational uncertainty, including systematic as well as measurement errors, would have to be reduced to 1 or 2% in many cases to make the distinction clear.

A potentially good indicator of SD1 *vs* SD2 is  $t_p \equiv P/\dot{P}$ . In conservative evolution this should be negative for SD1 systems and positive for SD2 systems. However the values are normally very uncertain, and often ambiguous. Periods can change by small amounts on a timescale of decades to centuries for many reasons, perhaps for several reasons simultaneously. As an example, Pribulla *et al.* (2000) in their analysis of the period history of RT And suggest that either the effect of a third body is superimposed on a long-term trend (with  $t_p$  as listed in Table 3) or else that there may be a fourth body producing the longer-term trend. We think that it is necessary to be very sceptical of any periodicity unless and until at least two whole periods have been covered; and even then one should be sceptical if the amplitude is not substantially greater than the scatter. Nevertheless we also feel sceptical of a  $t_p$  of the size indicated for RT And, since this seems to imply that angular-momentum loss (AML) is about 100 – 1000 times more powerful than nuclear evolution (NE) in this system.

Magnetic processes can cause cyclic, or quasi-cyclic, changes by a small amount, for example by causing both the moment of inertia and the quadrupole moment to fluctuate, perhaps by 1 part in  $10^4$  or so in decades. In addition, however, to the considerable uncertainty in the determination of  $t_p$ , the fact of strong magnetic activity suggests that a long-term period decrease cannot be equated automatically with an SD1 status; magnetic braking might manage to decrease the period of an SD2 system also. Further, magnetic

activity due to rapid rotation may cause mass loss (ML) which could potentially be on something like a nuclear timescale, and in that case the period could *increase*, even in an SD1 system.

Probably the best cases of SD1 systems are TT Her, V361 Lyr and VZ Psc. We can possibly add CN And to this list, since although  $X_1, X_2$  in Table 2 are very marginal the analysis was one of the most accurate, with the radial velocity curves solved simultaneously with the light curves, and allowance made for time-varying spots. van Hamme *et al.* (2001) found that several light-curves of CN And could all be fitted with almost the same fundamental parameters, plus two variable spots. They also found a period decrease, which tends to point in the direction of SD1 though not with great certainty as emphasised above.

The most convincing examples of SD2 systems are probably DI Peg, CX Aqr, YY Cet, W Crv and V386 Cyg. There seems little doubt that both SD1 and SD2 systems exist, and in comparable numbers despite the very different evolutionary histories that they imply.

Only a few NCBs have periods as short as the majority of LTCBs. The latter mostly have periods  $< 0.5$  d, but only five of the 25 NCBs listed (CN And, W Crv, BL Eri, FT Lup, V361 Lyr and VZ Psc) have similar periods. At least these five periods do span almost the whole range of periods seen in LTCBs. Although we feel that the masses in BL Eri may need revision, the period does not. A somewhat wider search, including systems with only single-lined orbits, and indeed with only light-curves, turns up about 25 probable NCB systems with  $P \leq 0.5$  d; 16 are listed by Shaw (1994). This is still at least an order of magnitude fewer than LTCB systems, if we also cast a comparably wider net there; and while 8 out of the 25 have  $P \leq 0.4$  d, the proportion among LTCBs is almost the reverse (40 out of 63 in Table 1).

Table 4. Means and standard deviations

	$\langle M \rangle$	$\langle \log q \rangle$	$\langle \log H \rangle$
DCB (11):	$2.07 \pm 0.53$	$-0.10 \pm 0.08$	$1.89 \pm 0.18$
NCB (25):	$2.27 \pm 0.72$	$-0.38 \pm 0.15$	$1.84 \pm 0.24$
LTCB (72):	$1.77 \pm 0.54$	$-0.50 \pm 0.24$	$1.54 \pm 0.28$
LTCB, W (42):	$1.66 \pm 0.41$	$-0.39 \pm 0.21$	$1.55 \pm 0.21$
LTCB, A (30):	$1.92 \pm 0.65$	$-0.64 \pm 0.27$	$1.53 \pm 0.34$

$M$  is the *total* mass,  $q$  is the mass ratio.

The unit of angular momentum  $H$  is  $M_{\odot} R_{\odot}^2/\text{day}$ .

Table 4 gives the mean and standard deviation of total mass, mass ratio and angular momentum in the three populations of Tables 1 – 3, and also in the sub-populations of A-type and W-type LTCBs. Recall that if a sample of  $N$  objects is drawn from a distribution with standard deviation  $\sigma$ , the *mean* of the sample has standard deviation  $\sigma/\sqrt{N}$  about the mean of the distribution.  $N$  is given in parentheses in the first column. Comparing the DCB and NCB samples, there is a significant difference in mean  $Q$ , but not in  $M$  or  $\log H$ ; for the NCB and LTCB samples, the latter has significantly smaller means for all three quantities, and the same is true if we compare the DCB and LTCB samples. That LTCBs have substantially lower angular momenta than DCBs and NCBs is partly because of the lower periods, as noted in the previous paragraph, and partly because of the smaller mass ratios.

The A-type and W-type sub-populations clearly differ substantially in mass ratio, as noted by Hilditch *et al.* (1988) and other authors. W-types appear to be less massive than A-types, although the difference is not strongly significant: about  $2\sigma/\sqrt{N}$ . Their angular momenta are not significantly different.

We should note that one condition ( $P < 1$  d) that we have used as a definition in all three samples is somewhat arbitrary as regards two of them. It is a reasonable cutoff for LTCBs, since indeed most of these have periods substantially less. But the population of NCBs merges into the population of normal Algols, several of which (e.g. R CMa) have periods of 1 – 2 d. That the shortest-period Algols can be classified as NCBs probably reflects the fact that the gainer is bound to be fairly near its Roche lobe in an orbit with  $P < 1$  d. As for DCBs, it is clear that these are simply the short-period tail of a distribution that increases

(per unit  $\log P$ ) fairly continuously but slowly up to  $\sim 100 - 200$  yr (Duquennoy & Mayor 1991, Halbwachs *et al.* 2003).

In anticipation of the next Section, we can ask whether there is any observational support in Table 4 for an evolutionary progression such as

$$DCB \rightarrow NCB \rightarrow LTCB(W) \rightarrow LTCB(A).$$

We can probably expect that all of NE, AML and ML play a role in at least some systems. NE should be unimportant in systems where both components are  $\lesssim 1 M_{\odot}$ , but note that NE may still have played a role in the past, or be capable of playing a role in the future, because of mass transfer. The drop in mass ratio between DCBs and NCBs may be a consequence of mass transfer, and fortunately the rise in total mass is not significant. However the fact that we cannot reliably distinguish observationally between SD1 and SD2 members of the NCB class limits the conclusions we can draw. An evolution from NCBs to LTCBs is also a possibility. If ML and AML apply at all, they can only reduce both  $M$  and  $H$ , but that is the sense observed, though only modestly. Once again the conflation of SD1s and SD2s among NCBs is not helpful.

The fact that mass-ratios are rather small for LTCBs is usually seen as implying that in contact binaries mass flows, on average, from the *less* to the *more* massive component. The alternative direction would suggest that LTCBs would crowd towards equal masses, which is very far from the case. The fact that the mass ratios decrease further from W-types to A-types might suggest further evolutionary progress in this direction, but we can say with some confidence that evolution from W-types as a whole to A-types as a whole is not supported, since the masses would have to *increase*. However, we cannot exclude the possibility that some (the less massive) A-types have evolved from W-types, while others (the more massive) have evolved more directly from NCBs. The lower mean mass of the W-types might indicate evolution (with ML but little AML) in the opposite direction, but then the increasing mass ratio would again lead us to expect a piling up of W-types at  $q \sim 1$ , contrary to what is seen.

#### 4. Evolutionary Processes

There are at least two very distinct ways of producing LTCBs, and several of producing NCBs (SD1 or SD2), starting from DCBs. We can in addition question (i) whether indeed all LTCBs and NCBs started from DCBs, or whether it is possible for the poorly-understood birth process of close binaries generally to produce LTCBs and/or NCBs *ab origine*, and (ii) whether, because of substantial angular-momentum loss and mass loss induced by rapid rotation, the NCBs might descend from typically wider and/or more massive systems than



the DCBs tabulated (which have an arbitrary cut-off at 1 d). However it is clear that many close binaries, not necessarily with  $P < 1$  d, *are* formed in the birth process, and it will be difficult for some of them to avoid evolving into RLOF and further into contact.

Nelson & Eggleton (2001) considered the conservative evolution of 5550 close (mostly Case A) pairs of stars, many of them in the range of masses and periods considered here. They identified 8 clearly distinguishable subCases of Case A, five of which led to contact. Two of particular relevance here are subCases AR (‘Rapid evolution to contact’) and AS (‘Slow evolution to contact’). Case AR occurs when the initial mass ratio is rather small ( $q_0 \lesssim 0.6$ ), and/or the period is so short that RLOF begins not long after the ZAMS. In this case the gainer swells up rather rapidly in response to accretion, while at the same time the orbit shrinks rather rapidly, so that contact is reached on a thermal timescale, and before much mass is transferred. An example of conservative Case AR evolution is given in Fig 4a ( $1.58 + 0.89 M_\odot$ , 0.56 d).

For Case AS we need a modest initial mass ratio (perhaps  $1 > q_0 \gtrsim 0.6$ ) and a somewhat longer period, up to roughly twice the period at which RLOF would begin on the ZAMS. In this case although the gainer swells it does not swell enough to reach its Roche lobe quickly, and instead RLOF continues as the mass ratio crosses from below to above unity. This leads to a fairly typical Algol, though of unusually short period. But the increase in mass of the gainer, coupled with the decrease of the loser, means that the gainer’s nuclear evolution can overtake the loser’s, and by the time the mass ratio has reached  $\sim 1.5 - 3$  the gainer has evolved to fill its own Roche lobe. An example of conservative Case AS evolution is given in Fig 4b ( $1.41 + 1.12 M_\odot$ ; 0.61 d).

There are Case A systems that can avoid contact, particularly subCase AN (‘Normal’). This needs a somewhat longer period than subCase AS, but a similar mild initial mass ratio ( $q_0 \gtrsim 0.6$ ). In this case the gainer, although its evolution is accelerated, is not able to catch up with the loser because the loser was already substantially evolved before RLOF began. This case generates normal Algols, in which the mass ratio increases from below unity to  $\sim 10$  (and the period also increases by an even larger factor) before the loser’s envelope is entirely lost and a proto-white-dwarf remnant is left. This is illustrated in Fig 4c ( $1.58 + 0.89 M_\odot$ ; 0.89 d).

The boundary between Cases AR and AS depends not just on period and mass ratio, but also on  $M_1$ . Below about  $1.2 M_\odot$  the ZAMS  $R, M$  relation is a good deal steeper than above (Fig 3a), and this means that Case AR is much more easily avoided in favour of Case AS. We see this in Fig 4d ( $1.0 + 0.68 M_\odot$ ; 0.4 d). The period is sufficiently short that we might expect Case AR here, but because of the steepness of the ZAMS  $R, M$  relation the RLOF is entirely on a nuclear timescale, and so the secondary does not swell up substantially as in Fig 4a. This shows that a DCB like XY UMa can in principle evolve to an SD2 NCB

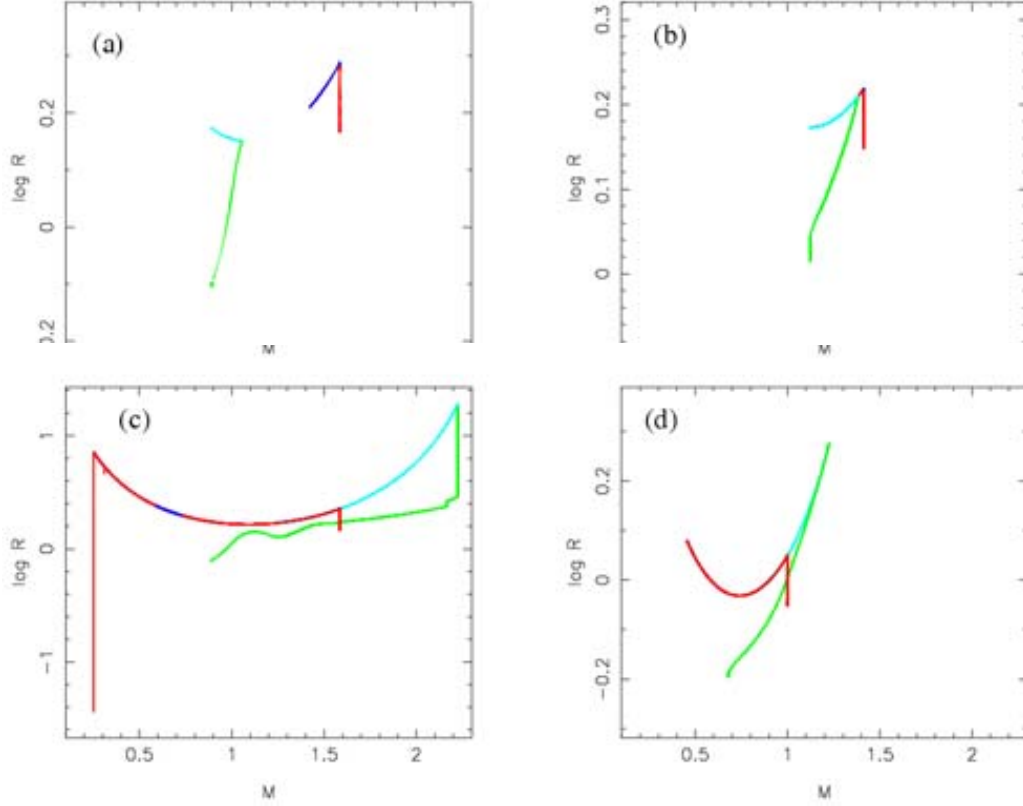


Fig. 4.— Conservative evolution of binaries in the  $\log R$  vs.  $M$  plane. Primary and secondary are red and green; their respective Roche-lobe radii are dark blue and light blue. Sometimes two or even three curves are superimposed. (a) Case AR,  $1.6 + 0.9 M_\odot$ , 0.56 d: once the primary starts RLOF, the secondary quickly grows to fill its own Roche lobe. (b) Case AS,  $1.4 + 1.1 M_\odot$ , 0.61 d: the secondary grows more slowly, and does not fill its Roche lobe until the mass ratio is almost reversed. (c) Case AN,  $1.6 + 0.9 M_\odot$ , 0.89 d: the secondary misses filling its Roche lobe until well after the primary has finished RLOF. The mass ratio at this point is  $\sim 10$ , and period  $\sim 14.3$  d. Subsequent evolution will probably involve reverse RLOF, a common-envelope stage, and a rapid merger to a single rapidly-rotating red giant. (d) A conservative Case AS track ( $1.0 + 0.7 M_\odot$ , 0.4 d) which evolves to a system very like W Crv once the initial mass ratio is reversed.

like W Crv on a timescale which is never faster than nuclear. However this evolution took  $\sim 12$  Gyr, and so we consider it unlikely that W Crv has in fact evolved conservatively by this route. To speed the evolution we would have to make the initial  $M_1$  substantially larger; but then we would have to make  $M_2$  smaller, and the more extreme mass ratio would almost certainly produce Case AR instead of Case AS evolution.

The above is all within the framework of *conservative* evolution, whereas in practice, for the rather low-mass and cool systems considered here, there is likely to be stellar wind and magnetic braking leading to ML and AML. If the AML timescale is shorter than the ML timescale, and also the NE timescale, this is likely to reduce the scope of Cases AS and AN and increase that of Case AR. For 8 of the 10 DCBs in Table 3, those with  $M_1 \lesssim 1.2 M_\odot$ , it seems quite probable that Case AR will be the major route to contact because of AML, and it may ensure that the primary is rather little evolved by the time RLOF, and then contact, is reached, even although the *present* periods are long enough in principle for Case AN or even Case B in some cases.

Although both Cases AR and AS suggest an evolutionary route leading from DCBs through NCBs to LTCBs, the NCB stage will be very different in the two cases: mainly SD1 in Case AR and mainly SD2 in Case AS. The former usually exist only on a thermal timescale, and the latter mainly on a nuclear timescale apart from a rapid phase in the beginning before the mass ratio is reversed. In addition the former are likely to have mass ratios fairly near unity, whereas the latter are likely to have mass ratios quite far from unity.

The above paragraph leads to the conclusion that SD2 systems should be *much* commoner than SD1 systems. This does not appear to be the case, although the statistics are very poor. Fortunately there is another route to SD1 systems. They may be a temporary, and repeated, phase in the evolution of LTCBs, via Thermal Relaxation Oscillations (TROs). TROs are a kind of evolution predicted by Lucy (1976), Flannery (1976) and Robertson & Eggleton (1977) for contact binaries.

It is somewhat surprising that there exists no clear understanding even of the structure, let alone the evolution, of LTCBs, although the subject has been addressed (fitfully) over the last 40 years or more. It is quite clear that there must be a substantial transfer of luminosity from one component to the other through the shallow contact envelope, in order to achieve the close equality of temperatures, since it is unreasonable to suppose that the two components just happen to have the same temperature despite very different masses. Models, including the TRO model (and the DSC model, see below) *assume* efficient transport, but no-one has yet put forward a plausible mechanism that will achieve this. Convection is an efficient form of heat transport, but it transports heat in the direction of gravity and what is required in LTCBs is heat transport *perpendicular* to gravity, i.e. roughly parallel to the surface. We

propose in Section 6 a new model that might solve this problem; but for the present we note that something like  $0.5 - 3 L_{\odot}$  of energy output from the more massive component, has to be transported ‘horizontally’ through the fairly narrow neck between the two stars (area  $\lesssim 0.1 R_{\odot}^2$ ), requiring horizontal fluxes there much greater than occur vertically in the Sun’s convective envelope.

Independently of the actual mechanism, however, one effect of efficient heat transport from the more to the less massive star (in the TRO model) is that the less massive star will tend to be expanded, and its surface raised to a higher gravitational-centrifugal potential than the companion. This is expected to lead to a net transfer of *mass*, back towards the companion. Such a transfer would tend to widen the orbit and lead to loss of contact. Then, with the opportunity for *luminosity* transfer removed, the primary now retaining its luminosity will be driven to expand, causing RLOF in an SD1 state. The secondary would initially contract, through losing its transferred luminosity, but will ultimately expand due to the increasing RLOF. This brings it back to contact, and the cycle can repeat.

In the longer term, the cyclic TRO evolution will gradually evolve because of NE and/or AML (and perhaps also ML). It will presumably evolve towards more extreme mass ratios, since if it evolved towards equal masses we would expect to see an accumulation of LTCBs at near-equal masses, and such an accumulation is completely at odds with what is observed.

A possible point of confusion is that we do not distinguish consistently between  $q > 1$  and  $q < 1$  in NCB (and LTCB) binaries. In Cases AS and AN  $q$  increases through unity to large values, whereas in Case AR  $q$  increases slightly and then (in the TRO model) decreases to small values. On either of routes AR and AS, during the LTCB phase  $q$  oscillates slightly about a slowly varying mean, during the TROs. We would make the distinction between  $q < 1$  and  $q > 1$  if we could, but (a) we have already shown that it is very hard to be certain in an NCB whether it is SD1 or SD2, and (b) we do not know of any way to distinguish the two classes of LTCB expected, those where the low-mass component is largely unevolved and those where it has had substantial evolution in its deep interior. It would be nice if the A/W dichotomy were due to this, but we suspect that the A/W dichotomy is more a consequence of which component happens to be more spotted.

Two interesting issues, as far as comparison with observation is concerned, are (a) the relative times spent in the DCB, NCB(SD1), NCB(SD2) and LTCB states, and (b) whether the populations of NCBs and LTCBs are sufficiently similar, in terms of mass, mass ratio and period, for the two to be as closely related as we expect. Robertson & Eggleton (1977) estimated crudely that the time in the contact part of a TRO is dictated by the thermal timescale of the secondary, while that in the SD1 state is dictated by the thermal timescale of the primary. If the two stars in the NCB state were more-or-less ZAMS stars, the two

timescales would be very different, and would favour systems being predominantly LTCBs. However the secondary is considerably different from a ZAMS star, and the two timescales may well be within a factor of 10 of each other even though the mass ratio (in LTCBs) is typically  $\sim 0.3 \pm 0.2$ . The best we can say at the moment is that we might expect the ratio of NCB(SD1) systems to LTCBs to be of order 10%. Neither sample, in Tables 1 and 2, is near complete, either in magnitude or in distance, but this expected ratio is not manifestly at odds with the observations.

What is needed in addition to a model for heat and mass transfer in contact is a model for AML (and ML, which is bound to go along with it) that gives the the stellar wind strength and the Alfvén radius as functions of the following four parameters of a star: mass, radius, luminosity and rotation rate. An intermediate quantity, the Rossby number (rotation period divided by convective envelope turnover time), is widely thought to be important, but in principle this is a function of the four parameters listed. Eggleton (2001) proposed such a model: see next Section. It is bound to be a gross over-simplification, but for the present there is no soundly-based MHD theory which accounts quantitatively for dynamo action in turbulent, convective, rotating (and differentially rotating) stellar interiors, and which relates the dynamo action to the strength of stellar wind, and to the Alfvén radius.

Shu et al (1976) suggested a structure for contact binaries that in principle is rather different from TROs. They *postulated* that stars in contact develop a temperature discontinuity  $\Delta T$  below the shared envelope in the less massive component (the DSC model), and that the magnitude of this discontinuity would adjust itself to be such that there would be no (net) mass transfer between the components. In other words, by introducing a free parameter  $\Delta T$  they had enough parameters to be able to construct a model in complete equilibrium for a given  $M_1$  and  $M_2$ . In the TRO model this parameter is absent, and its place is effectively taken by a net mass transfer rate  $\dot{M}_1 = -\dot{M}_2$ . This  $\dot{M}_1$  must be on a thermal timescale if it is to be large enough to satisfy the constraints of equal surface potential *and* equal temperatures. Because the DSC model assumes that  $\dot{M}_1 = \dot{M}_2 = 0$ , it does not predict any mass change on a thermal timescale, and in fact it does not make any prediction regarding mass transfer in response to the slower but inevitable evolution on NE or AML timescales. For any two masses, period and internal evolutionary states it predicts a  $\Delta T$  that keeps the masses constant. This seems to contradict the observational fact that LTCBs normally have rather small mass ratios whereas DCBs normally do not.

## 5. Some Non-Conservative Evolutionary Models

It might seem, and indeed is, straightforward to run a grid of non-conservative models analogous to the grid of conservative models run by Nelson & Eggleton (2001); at any rate it is if, in the absence of a good model of heat transport during the contact phase, we simply stop the computation at the point when contact is reached. However, it is somewhat unlikely that the specific non-conservative model of Eggleton (2001) would give the right answers without modification, and if one allows for certain parameters in the model to take a range of plausible values then the amount of modeling grows astronomically.

We will not repeat the mathematical description of the model, which is given more fully in Eggleton & Kiseleva-Eggleton (2002): we refer to it as the EKE02 model. In essence,

- (a) from  $M, R$  and  $L$  we estimate the convective envelope turnover time, and hence the Rossby number from the period;
- (b) this leads to an estimate of both the differential rotation rate (crucial for dynamo activity) and the length of the magnetic cycle ( $\sim 11$  yr for the Sun);
- (c) we estimate both the mean poloidal field (crucial for the Alfvén radius) and the mean toroidal field (crucial for driving stellar wind);
- (d) we estimate the strength of the wind,  $\dot{M}_i$  ( $i = 1, 2$ ), assuming that the dissipation of the toroidal field in the course of a magnetic cycle provides the driving energy;
- (e) then the combination of wind strength and poloidal field gives the Alfvén radius and thus the rate of AML,  $\dot{H}_i$  ( $i = 1, 2$ );
- (f) a model of tidal friction must also be incorporated, so that spin angular momentum can be transformed into orbital angular momentum, and *vice versa*. Of course for the short periods considered here one could reasonably suppose that stars and orbits are always synchronised, but the model proposed ought to be able to be applied to all binaries, including those which are initially fairly wide and possibly eccentric.

Note that the  $\dot{M}_i$  in (d) refer to wind only; the total  $\dot{M}_i$  of a component will also include some mass transfer, in SD and C systems.

This code has had some modest success in dealing with rather wider binaries (RS CVn systems, normal Algols; EKE02) but has not yet been tested on systems rotating typically ten times faster. We should note that whereas EKE02 applied the ML/AML equations to only one of the two components, a simplification which makes the calculation a great deal easier and which is reasonably valid for ordinary Algols where only one component (the loser) is cool enough for significant dynamo activity, here we use a version of the code which

allows both components to be solved *simultaneously*. Thus both components can be active stars at the same time, something which seems necessary since, according to Table 2, both components are frequently later than  $\sim F0$ , where empirically stellar activity due to rapid rotation is found.

We consider here only a few cases, starting with roughly the mean total mass and initial mass ratio of the DCBs, i.e. masses  $1.18 + 0.94 M_{\odot}$ . We take initial periods between 0.4 d, slightly more than the period at which the primary fills its Roche lobe while still on the ZAMS, and  $\sim 2$  d. Fig 5a ( $P_0 = 0.75$  d) is fairly typical of the whole range of periods. The system evolves primarily because of ML and AML, with rather little NE. In  $\sim 2.2$  Gyr the system shrinks to RLOF, and in another  $\sim 0.1$  Gyr reaches contact. The masses decrease by  $\sim 20\%$  and  $\sim 10\%$  until RLOF, but then the secondary gains by accretion roughly what it lost by enhanced stellar wind. The main effect of various initial periods is that the process takes longer at longer period, not surprisingly, and requires a Hubble time at  $\gtrsim 2.7$  d. The evolution is essentially Case AR. In the semidetached phase, the system resembles VZ Psc, though at slightly higher masses.

The primary, though not evolved substantially, is slightly evolved, and is  $\sim 5\%$  larger than its ZAMS radius when RLOF begins. This might contribute to the fact that in Fig 3a the DCBs are mostly a little above the ZAMS; but it cannot be the whole story since we would still expect the *secondaries* to be close to the ZAMS.

It is a feature of the EKE02 model that, although both the magnetic field and the wind increase with rotation rate (but reaching saturation as the Rossby number becomes small, i.e. the rotation is rapid), the Alfvén radius can *decrease* at rapid rotation because the high density of a strong wind – one with an ML timescale of  $\sim 10$  Gyr – makes the Alfvén radius fairly small, despite a strong magnetic field. On the ZAMS and at rotation periods of  $\sim 1$  d, the ML timescale is computed to be comparable to the NE timescale around  $\sim 1 M_{\odot}$ , and the AML timescale somewhat *longer*. But going down the ZAMS, at the same rapid rotation rates, the ML timescale lengthens, thus increasing the Alfvén radius, so that below about  $0.8 M_{\odot}$  AML is the only one of the three timescales that is significant in a Hubble time.

Fig 5b is the rather more complex evolution of a system with somewhat greater initial masses and period  $1.9 + 1.4 M_{\odot}$ ; 0.94 d. NE is slightly faster than ML to start with, and the system appears to be heading fairly directly for RLOF. But as the primary becomes larger and cooler, it becomes more active, and ML begins to dominate NE so that the primary starts shrinking again. Its Roche lobe also shrinks, at almost exactly the same rate; the primary comes within a whisker of RLOF ( $R/R_L \sim 0.99$ ) when its mass is reduced to  $\sim 0.5 M_{\odot}$ , but then fractionally retreats. We quite commonly find this to be a fairly stable process, with ML self-adjusting so that the star stays just inside its Roche lobe while its mass decreases,

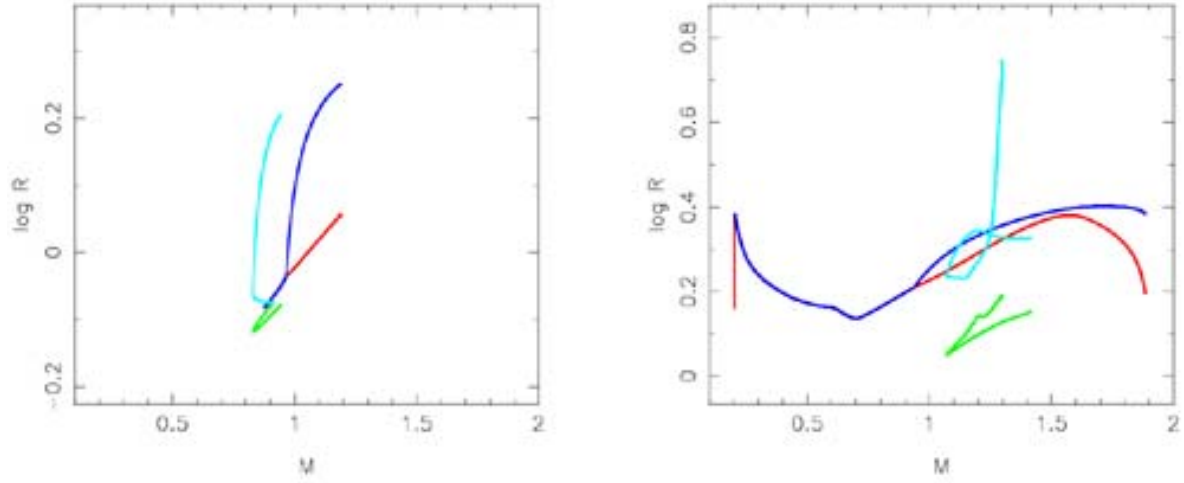


Fig. 5.— Non-conservative evolution of binaries in the  $\log R$  vs.  $M$  plane. Primary and secondary are red and green; their respective Roche-lobe radii are dark blue and light blue. Sometimes two curves are superimposed. (a) Initial parameters  $1.19 + 0.94 M_{\odot}$ ; 0.75 d: angular-momentum loss dominates, and the system shrinks. It reaches RLOF at  $0.97 + 0.83 M_{\odot}$ ; 0.31 d, and contact at  $0.88 + 0.91 M_{\odot}$ ; 0.28 d. The original primary is only slightly evolved at this point. (b) Initial parameters  $1.88 + 1.41 M_{\odot}$ ; 0.94 d. As the primary expands and becomes cooler, mass loss becomes significant, and after the radius peaks both the star and its Roche lobe shrink, at almost the same rate. RLOF is not reached until  $0.94 + 1.07 M_{\odot}$ ; 0.76 d. Angular-momentum loss causes the system to shrink until  $0.72 + 1.15 M_{\odot}$ ; 0.69 d, but then the core of the primary reaches hydrogen exhaustion, and nuclear evolution becomes dominant. The system expands until RLOF ceases at  $0.20 + 1.30 M_{\odot}$ ; 3.12 d. The later stages are like a normal Algol, but of unusually low mass and short period.



perhaps as much as by a factor of two. But at  $\sim 2.0$  Gyr, with the primary reduced to  $0.94 M_{\odot}$ , the star finally reaches its lobe, and RLOF continues for  $\sim 0.9$  Gyr. However the secondary accretes only a small amount of mass: most of the mass leaving the primary is wind rather than an accreting stream.

To be more precise, although the secondary accretes from the primary, it also loses mass by an enhanced wind as does the primary, and the timescales of NE and of ML from *both* components, are comparable. No doubt this phase involves rather complicated gas dynamics, as well as MHD, and so the modeling is particularly tentative here.

When the primary reaches central hydrogen exhaustion, however, NE becomes faster than ML, though only by a modest factor. The binary begins to expand, like a conventional Algol, and the primary is stripped to its helium core of  $\sim 0.2 M_{\odot}$  while the orbital period increases, from 0.69 to 3.12 d. During this phase, when the period is  $\sim 1.2$  d, the system quite closely resembles the Algol R CMa ( $0.17+1.07 M_{\odot}$ ; 1.14 d; Sarma *et al.* 1996), a system which just falls outside the period range of Table 2. R CMa must clearly have lost substantial mass and angular momentum, as well as having undergone considerable nuclear evolution. Although we can by no means claim that the EKE02 model uniquely explains this system, it is clear that the substantial mass loss that the EKE02 model produces is not unreasonable.

At a somewhat earlier stage, when  $P \sim 0.75$  d, the system in Fig 5b would have resembled CX Vir, in Table 2; even earlier, at its minimum period of  $\sim 0.69$  d, it would have resembled CX Aqr. It seems reasonable to suppose that several of the longer-period NCBs may have a somewhat similar evolution. It may therefore be the case that several NCB(SD2) systems do *not* evolve into contact, but instead evolve like normal Algols but with unusually low mass and angular momentum. Possibly this is at least part of the answer to the problem that there are hardly any contact binaries with periods in excess of 0.5 d, despite the fact that there are several NCBs in this range.

We represent non-conservative versions of Cases AR, AS (including TROs) and AN by the following evolutionary chains:

AR : DCB(AML, NE, ML)  $\rightarrow$  NCB, SD1(TH)  $\rightarrow$

$$\rightarrow \{\text{LT CB(TH)} \leftrightarrow \text{NCB, SD1(TH)}\}(\text{AML, ML, NE}) \rightarrow \text{merger } (q \rightarrow 0), \quad (5)$$

AS : DCB(NE, AML, ML)  $\rightarrow$  NCB, SD1(TH)  $\rightarrow$  NCB, SD2(NE, AML, ML)  $\rightarrow$

$$\rightarrow \{\text{LT CB(TH)} \leftrightarrow \text{NCB, SD1(TH)}\}(\text{AML, NE, ML}) \rightarrow \text{merger } (q \rightarrow \infty), \quad (6)$$

AN : DCB(NE, ML, AML)  $\rightarrow$  DCB(ML, NE, AML)  $\rightarrow$  NCB, SD2(ML, NE, AML)  $\rightarrow$

$$\rightarrow \text{DEB(NE, ML)} \rightarrow \text{reverse RLOF} \rightarrow \text{common envelope} \rightarrow \text{merger}. \quad (7)$$

In this notation various timescales are indicated in parentheses: NE for nuclear evolution, TH for thermal evolution, AML for angular-momentum loss by magnetic braking, and ML for mass loss by binary-enhanced stellar wind. Where two or more timescales are listed in the same parentheses, we suggest that all may be significant but the first is probably more significant than the second, etc. In route (7), DEB stands for ‘detached, evolved binary’, where the remains of the primary is a hot subdwarf-B star.

Few would argue against the proposition that AML can be competitive with NE in driving the evolution of DCB, NCB and LTCB systems. The proposition that ML is competitive with both of them is probably more controversial. But the fact that three NCB,SD2 systems in Table 2, CX Aqr, CX Vir and W Crv, have total masses which are so low that it is difficult to see how NE could ever have been important in them makes it seem quite plausible that some systems may have lost  $\gtrsim 20\%$  of their total mass. While AML leads to orbital shrinkage, ML can lead to orbital expansion. The EKE02 model, without being designed specifically to do so, makes all three processes competitive in stars with masses  $\sim 1 - 1.5 M_{\odot}$  that are rotating with  $P \sim 0.5 - 1$  d. This leads to a somewhat embarrassing wealth of possibilities for evolution, and makes it particularly difficult to predict the fraction of initial systems that will evolve in various ways.

## 6. A New Model of Heat Transport in Contact Binaries

We mentioned above that the near-equality of temperatures in contact binaries is still mysterious: it demands a *very* efficient means of transporting heat, and one that unlike convection does not require gravity, at least directly. We suggest that the mechanism may be *differential rotation*, as observed by helioseismology in the Solar convection zone (Schou *et al.* 1999).

The Sun has an equatorial belt, as deep as the surface convection zone (the outer 30% by radius) and  $\sim \pm 30^\circ$  wide, which is rotating about 10% faster than the mean rotation of the Sun; it rotates about once per year faster than the mean Sun. This means that it is carrying (advecting, rather than convecting) an enormous heat flux sideways, i.e. horizontally. The ratio of heat advected sideways to heat convected radially is roughly  $(\Delta E/E_{\text{th}}) \cdot \Delta\Omega t_{\text{th}}$ , where  $\Delta E$  is the thermal energy in the convection zone,  $E_{\text{th}}$  is the thermal energy of the whole Sun,  $\Delta\Omega$  is the differential rotation rate ( $\sim 2\pi \text{ yr}^{-1}$ ) and  $t_{\text{th}} \sim 5 \cdot 10^7 \text{ yr}$  is the thermal timescale of the whole Sun. A crude estimate of  $\Delta E/E_{\text{th}}$  is  $\sim 10^{-5}$ . The ratio of horizontal to vertical flux is therefore  $\sim 3000$ .

This huge horizontal flux has, of course, no discernible effect on the Sun’s appearance, because the Sun is closely axisymmetric. But if the Sun had a companion, say half its mass, in contact with it, then the differential rotation, no doubt driven as much by the companion as by the Sun, could carry this heat from one body to the other and back, and result in making the joint surface much more nearly uniform than it would be if the stars were not in contact.

The typical depth of contact in an LTCB is not 30% of the radius, but more like 3 – 8%: see  $X_1, X_2$  in Table 1. This reduces  $\Delta E$  quite substantially, since the density decreases quite rapidly outwards and the temperature fairly rapidly. But if  $\Delta E$  is reduced by  $\sim 300$ , we still have a considerable amount of sideways heat transport.

It is not yet clear just how the solar differential rotation is driven, but it is generally suspected to be a result of the combination of turbulent convection and Coriolis force. Much the same combination may drive (a) various atmospheric currents on Earth (the trade winds, the jet stream), and (b) belts of positive and negative differential rotation on Jupiter and other massive, gaseous planets. We suggest that differential rotation is a fairly universal result of the combination of turbulent convection and rotation. The EKE02 model includes it as part of the dynamo model, and predicts that whereas differential rotation is  $\sim 10\%$  in the Sun, it is more like  $\sim 0.1 - 1\%$  in very rapidly rotating stars (and also in very slowly rotating stars, like red giants). At  $\sim 1\%$  it is still likely to be strong enough to carry the necessary heat from one component to the other in a contact binary.

Fig 6 is a preliminary model of evolution into and during contact, in which the luminosity transfer is modeled by a sink (or source, in the less massive companion) of energy generation given by

$$\epsilon = \pm \Delta\Omega (h_2 - h_1) \quad . \quad (8)$$

The  $h$ ’s are the enthalpies in the two components at approximately the same potential, provided that this potential is above the potential of the inner critical lobe.  $\Delta\Omega$ , the differential rotation rate, was taken to be  $10^{-8}/\text{s}$ , less than the Solar value and  $\sim 2 \cdot 10^{-4}$  of the binary rotation rate. Initial parameters were  $(1.26 + 1.0 M_\odot; 0.47 \text{ d})$ . This system evolved marginally by Case AS rather than AR, with contact first being reached at parameters  $(1.04 + 1.11 M_\odot; 0.38 \text{ d})$ . After two or three TROs, the parameters settled down to an oscillation of about 0.2% amplitude about the values  $(0.90 + 1.22 M_\odot; 0.40 \text{ d})$ . Fig 6 shows that the oscillations in luminosity are substantially greater, about 10%. There appears to be a slight tendency for the masses to become more unequal over several TROs, as expected on account of AML, but the run is not long enough to be definitive.

Fig 6 plots the luminosities of the two components in solar units, and also the luminosity transfer during the contact portions of the subsequent TROs. In the first plot the

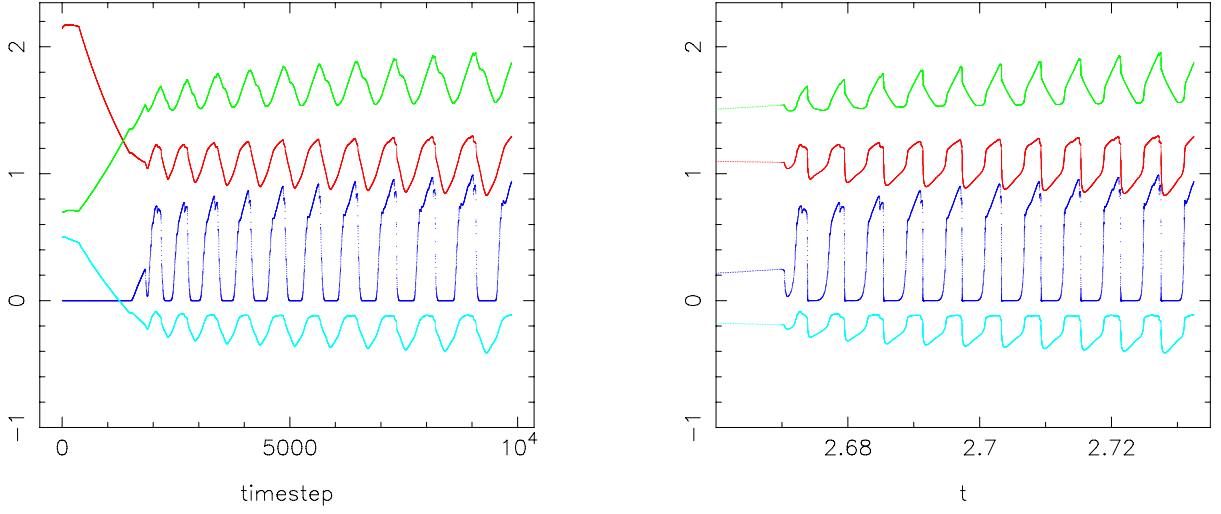


Fig. 6.— The luminosity of the primary (red) and of the secondary (green); the amount of luminosity transferred (dark blue), and the difference in  $\log_{10} T$  multiplied by 10 (light blue). The luminosities are in solar units. The abscissa is timestep number (left panel) and time in Gyr (right panel). Once TROs begin, their cycle time is about 60 Myr. The luminosities, and also the masses, passed through equality during the semidetached phase preceding the TROs.

independent variable is timestep number; we use this rather than time because all of the oscillations occur in a relatively short period of time at the end of the computed evolution. This is illustrated in the second plot, where the 10 oscillations computed occur between ages 2.67 and 2.73 Gyr. Also shown is the difference in  $\log_{10} T$  between the two components, multiplied by 10 to make it clearer. This difference is about 0.01 during the contact part of the cycle, about 40% of the cycle, and peaks at about 0.04 during the remaining 60%. In contact, the temperature difference is about right for A-type systems; in the SD state the temperature difference is rather large for apparent contact systems, and rather small for NCB systems. However the mass ratio of the model is very mild compared with most LTCB and NCB systems, and this may explain why the temperatures differ only by  $\sim 10\%$  in the SD phase.

The masses and temperatures of the model in contact are rather similar to those of OO Aql (Table 1), but OO Aql has a longer period (0.5 d) which suggests more nuclear evolution, consistent with a longer initial period. The masses are also somewhat like those of V361 Lyr (Table 2), although our model in the SD1 state of a TRO has more nearly equal temperatures. V361 Lyr also has a *shorter* period (0.31 d), and presumably *less* nuclear evolution.

It will be a major computational task to attempt to model the generality of LTCB and NCB binaries, and endeavour to fit them all into a comprehensive scheme. This is because (a) the evolution of a single case involves several hundred times more computation than for the case (itself not easy) where contact does not occur, and (b) it is not clear *a priori* that the highly simplistic non-conservative model used here (EKE02) is even approximately right, and it would be desirable at the very least to introduce into it some free parameters whose values might conceivably be determined by goodness of fit. In addition, it is likely that (c) the computational difficulty becomes greater the more extreme is the mass ratio: this is the reason why our preliminary model has a much milder mass ratio than we would like. Nevertheless we feel that it would be a very worthwhile task, since we believe that the evolution of contact binaries is one of the three most outstanding theoretical problems in binary-star research. The others, we suggest, are (i) the formation of close binaries in the first place, and (ii) the evolution into and out of the common-envelope stage of Paczyński (1976).

A 3D code called ‘Djehuty’ (Bazan *et al.* 2003, Eggleton *et al.* 2003) is being developed at the Lawrence Livermore National Laboratory, which will be capable of modeling entire stars. This code does not yet have the capability to resolve the small-scale motion expected in the Sun’s outer layers, but the next generation of massively-parallel computers should be capable of tackling this problem, and consequently also the problem of shared envelopes in

LTCBs. 3D modeling is of course very much slower than 1D modeling, but we can expect that by studying the structure in the course of a few rotations of the binary we can gain insight into the validity of a model such as equation (8) for heat transport, and so parametrise more reliably a 1D approximation that could work over much longer timescales.

## 7. Acknowledgments

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## REFERENCES

- Ahn, Y. S., Hill, G. & Khalessheh, B., 1992 (A92), A&A, 265, 597 (A92)
- Albayrak, B., Djurasevic, G., Erkapic, S. & Tanrverdi, T., 2004 (B04), A&A, 420, 1039
- Aslan, Z., Özdemir, T., Yeşilyaprak, C. & İskender, A., 1999, Tr. Journal of Physics, 23, 45.
- Awadalla, N. S. & Yamasaki, A., 1984 (A84), Ap&SS, 107, 347.
- Awadalla, N. S., 1989 (A89), Ap&SS, 162, 211.
- Baran, A., Zola, S., Rucinski, S. M., Kreiner, J. M., Siwak, M. & Drozd, M., 2004 (B04a), ActaA, 54, 195.
- Barnes, J. R., Lister, T. A., Hilditch, R. W. & Collier Cameron, A., 2004 (B04b), MNRAS, 348, 1321.
- Barone, F., di Fiore, L., Milano, L. & Russo, G., 1993 (B93), ApJ, 407, 237.
- Bazan, G., Cavallo, R. M., Dearborn, D. S. P., Dossa, D. D., Keller, S. C., Taylor, A. G. & Turcotte, S., 2003, in *3-D Stellar Evolution* eds Turcotte, S., Keller, S. C. & Cavallo, R. M. ASP conf. 293, p15
- Bell, S. A., Rainger, P. P. & Hilditch, R. W., 1990 (B90a), MNRAS, 247, 632.
- Bell, S. A., Rainger, P. P., Hill, G. & Hilditch, R. W., 1990 (B90b), MNRAS, 244, 328.
- Binnendijk, L., 1970, Vistas in Ast., 12, 217

- Budding, E., Butler, C. J., Doyle, J. G., Etzel, P. B., Olah, K., Zeilik, M. & Brown, D., 1996 (B96), *Ap&SS*, 236, 215.
- Cester, B., Mardirossian, F. & Pucillo, M., 1977 (C77), *A&A*, 56, 75.
- Chochol, D., van Houten, C. J., Pribulla, T. & Grygar, J., 2001 (C01), *CoSka*, 31, 5.
- Corcoran, M. F., Siah, M. H. & Guinan, E. F., 1991 (C91), *AJ*, 101, 1828.
- Covino, E., Barone, F., Milano, L., Russo, G. & Sarna, M. J., 1990 (C90), *MNRAS*, 246, 472.
- Csák, B., Kiss, L. L., Vink, J. & Alfaro, E. J., 2000 (C00), *A&A*, 356, 603
- Derekas, A., Kiss, L. L. & Bebesi, Z. S., 2002 (D02), *IBVS* 5255.
- Duerbeck, H. W. & Karimie, M. T., 1979 (D79), *IBVS*, 3305.
- Duerbeck, H. W. & Schumann, J. D., 1982 (D82), *JAA*, 3, 233.
- Duquenooy, A. & Mayor, M., 1991, *A&A*, 248, 485.
- Eggleton, P. P., 1981, *SAOSR*, 392, 153.
- Eggleton, P. P., 1983, *ApJ*, 268, 368.
- Eggleton, P. P., 1996, in *The Origins, Evolution and Destinies of Binaries in Clusters* ed. Milone, E. F. & Mermilliod, J.-C., *ASP Conf series*, 90, 257
- Eggleton, P. P., 2001, in *Evolution of Binary and Multiple Star Systems* ed. Podsiadlowski, Ph., Rappaport, S., King, A. R., D’Antona & Burderi, L. *ASP Conf. Ser* 229, 157.
- Eggleton, P. P., Bazan, G., Dearborn, D. S. P., Dossa, D. D., Taylor, A. G., Castor, J. I., Murray, S., Cook, K. H., Eltgroth, P. G., Cavallo, R. M., Turcotte, S., Keller, S. C. & Pudliner, B. S., 2003, in *3-D Stellar Evolution* eds Turcotte, S., Keller, S. C. & Cavallo, R. M. *ASP conf.* 293, p1
- Eggleton, P. P. & Kiseleva-Eggleton, L., 2002 (EKE02), *ApJ*, 575, 461.
- Flannery, B. P., 1976, *ApJ* 205, 217.
- Frederik, M. C. G. & Etzel, P. B., 1996 (F96), *AJ*, 111, 2081.
- Goecking, K.-D., Duerbeck, H. W., Plewa, T., Kaluzny, J., Schertl, D., Weigelt, G. & Flin, 1994 (G94), *A&A*, 289, 827.

- Goecking, K.-D., and H. W. Duerbeck, 1993 (G93), A&A, 278, 463.
- Guinan, E. F. & Koch, R. H., 1977 (G77), PASP, 89, 74.
- Halbwachs, J. L., Mayor, M., Udry, S. & Arenou, F., 2003, A&A, 397, 159.
- Hazlehurst, J. & Refsdal, S., 1978, A&A, 62, 177.
- Hilditch, R. W., Collier Cameron, A., Hill, G., Bell, S. A. & Harries, T. J., 1986, (H86a) MNRAS, 223, 607.
- Hilditch, R. W., Collier Cameron, A., Hill, G., Bell, S. A. & Harries, T. J., 1997 (H97), MNRAS, 291, 749.
- Hilditch, R. W. & King, D. J., 1986 (H86b), MNRAS, 223, 581.
- Hilditch, R. W. & King, D. J., 1988 (H88a), MNRAS, 231, 397.
- Hilditch, R. W. & King, D. J., 1988 (H88b), MNRAS, 232, 147.
- Hilditch, R. W., King, D. J., Hill, G. & Poeckert, R., 1984 (H84), MNRAS, 208, 135.
- Hilditch, R. W., King, D. J. & McFarlane, T. M., 1988 (H88c), MNRAS, 231, 341.
- Hilditch, R. W., King, D. J. & McFarlane, T. M., 1989 (H89), MNRAS, 237, 447.
- Hilditch, R. W., Hill, G. & Bell, S. A., 1992 (H92), MNRAS, 255, 285.
- Hrivnak, B. J., 1988 (H88d), ApJ, 335, 319.
- Hrivnak, B. J., Guinan, F. E., DeWarf, L. E. & Ribas, I., 2001 (H01), AJ, 121, 1084.
- Hrivnak, B. J., Guinan, E. F. & Lu, W., 1995 (H95), ApJ, 455, 300.
- Kähler, H., 2002a, A&A, 395, 889.
- Kähler, H., 2002b, A&A, 395, 907.
- Kähler, H., 2004, A&A, 414, 317.
- Kalçı, R. & Derman, E., 2004 (K04a), private comm.
- Kałużny, J., 1984 (K84a), AcA, 34, 217.
- Kałużny, J., 1985 (K85), AcA, 35, 327.
- Kałużny, J. & Rucinski, S. M., 1986 (K86), AJ, 92, 666.



- Khajavi, M., Edalati, M. T. & Jassur, D. M. Z., 2002 (K02a), *Ap&SS*, 282, 645.
- Kim, C.-H., Lee, J. W., Kim, S.-L., Han, W. & Koch, R. H., 2003 (K03a), *AJ*, 125, 322.
- King, D. J. & Hilditch, R. W., 1984 (K84b), *MNRAS*, 209, 645.
- Kjurkchieva, D. P., Marchev, D. V., Heckert, P. A. & Shower, C. A., 2004 (K04b), *A&A*, 424, 993.
- Kjurkchieva, D. P., Marchev, D. V. & Ogłóza, W., 2004 (K04c), *A&A*, 415, 231.
- Kjurkchieva, D. P., Marchev, D. V. & Zoła, S., 2002 (K02b), *A&A*, 386, 548.
- Kjurkchieva, D. P., Marchev, D. V. & Zoła, S., 2003 (K03b), *A&A*, 404, 611.
- Kreiner, J. M., Rucinski, S. M., Zoła, S., Niarchos, P., Ogłóza, W., Stachowski, G., Baran, A., Gazeas, K., Drozd, M., Zakrzewski, B., Pokrzywka, B., Kjurkchieva, D. & Marchev, D., 2003 (K03c), *A&A*, 412, 465.
- Lapasset, E. & Gomez, M., 1990 (L90), *A&A*, 231, 365.
- Li, L., Han, Z. & Zhang, F., 2004, *MNRAS*, 351, 137.
- Lipari, S. L. & Sisteró, R. F., 1986 (L86), *MNRAS*, 220, 883.
- Lipari, S. L. & Sisteró, R. F., 1987 (L87), *AJ*, 94, 792.
- Lu, W., 1992 (L92), *ActaA*, 42, 73.
- Lu, W. & Rucinski, S. M., 1993 (L93), *AJ*, 106, 361.
- Lu, W. & Rucinski, S. M., 1999 (L99), *AJ*, 118, 515.
- Lu, W., Rucinski, S. M. & Ogłóza, W., 2001 (L01), *AJ*, 122, 402.
- Lubow, S. H. & Shu, F. H., 1977, *ApJ*, 216, 517.
- Lubow, S. H. & Shu, F. H., 1979, *ApJ*, 229, 657.
- Lucy, L. B., 1976, *ApJ*, 205, 208.
- Lucy, L. B. & Wilson, R. W., 1979, *ApJ*, 231, 502.
- Maceroni, C., Milano, L. & Russo, G., 1984 (M84), *A&AS*, 58, 405.
- Maceroni, C., Milano, L. & Russo, G., 1985, *MNRAS*, 217, 843.

- Maceroni, C. & van't Veer, F., 1996, *A&A*, 311, 523.
- Maceroni, C., Vilhu, O., van't Veer, F. & van Hamme, W., 1994 (M94), *A&A*, 288, 529.
- McFarlane, T. M., Hilditch, R. W. & King, D. J., 1986 (M86a), *MNRAS*, 223, 595.
- McFarlane, T. M., King, D. J. & Hilditch, R. W., 1986 (M86b), *MNRAS*, 218, 159.
- McLean, B. J. & Hilditch, R. W., 1983 (M83), *MNRAS*, 203, 1.
- Mannino, G., 1958 (M58), *MSIA*, 29, 433.
- Maxted, P. F. L. & Hilditch, R. W., 1996 (M96), *Obs*, 116, 288.
- Metcalfe, T. S., 1999 (M99), *AJ*, 117, 2503.
- Milone, E. F., Wilson, R. E. & Hrivnak, B. J., 1978 (M87), *ApJ*, 319, 325.
- Milano, L., Barone, F., Mancuso, S., Russo, G. & Vittone, A.A., 1989 (M89), *A&A*, 210, 181.
- Milone, E. F., Stagg, C. R., Sugars, B. A. & McVean, J. R., 1995 (M95)
- Mochnecki, S. W., 1981, *ApJ*, 245, 650.
- Mochnecki, S. W., Fernie, J. D., Lyons, R., Schmidt, F. H. & Gray, R. O., 1986 (M86c), *AJ*, 91, 1221.
- Nelson, R. H., Milone, E. F., Vanleeuwen, J., Terrell, D., Penfold, J. E. & Kallrath, J. 1995 (N95), *AJ*, 110, 240.
- Nelson, C. A. & Eggleton, P. P., 2001, *ApJ*, 552, 664.
- Nesci, R., Maceroni, C., Milano, L. & Russo, G., 1986 (N86), *A&A*, 159, 142.
- Niarchos, P. G. & Manimanis, V. N., 2003 (N03), *A&A*, 405, 263.
- Nomen-Torres, J. & Garcia-Melendo, E., 1996 (N96), *IBVS*, 4365
- Odell, A. P., 1996 (O96), *MNRAS*, 282, 373.
- Oh, K.-D. & Ahn, Y.-S., 1992 (O92), *Ap&SS*, 187, 261.
- Ovenden, M. W., 1954 (O54), *MNRAS*, 114, 569.
- Özdemir, S., Demircan, O., Cicek, C. & Erdem, A., 2004 (O04), *AN*, 325, 1.

- Qian, S. & Yang, Y., 2004 (Q04), *AJ*, 128, 2430
- Qian, S. & Yang, Y., 2005 (Q05), *MNRAS*, 356, 765
- Paczynski, B., 1976 in *Structure and Evolution of Close Binary Systems*, eds Eggleton, P. P., Whelan, J. A. J. & Mitton, S. M., *IAU Symp*, 73, 75
- Pazhouhesh, R. & Edalati, M. T., 2002 (P02a), *IBVS*, 5236.
- Pearce, J. A., 1933 (P33), *JRASC*, 27, 62.
- Pojmański, G., 1998 (P98), *ActaA*, 48, 711.
- Pols, O. R., Tout, C. A., Eggleton, P. P. & Han, Z., 1995, *MNRAS*, 274, 964.
- Popper, D. M., 1980 (P80), *ARA&A*, 18, 115.
- Popper, D. M., 1988 (P88), *AJ*, 95, 190.
- Popper, D. M., 1994 (P94), *AJ*, 108, 1091.
- Popper, D. M., 1997 (P97), *AJ*, 114, 1195.
- Pribulla, T., Chochol, D., Rovithis-Livaniou, H. & Rovithis, P., 1999 (P99), *A&A*, 345, 137.
- Pribulla, T., Chochol, D., Milano, L., Errico, L., Vittone, A. A., Barone, F. & Parimucha, Š., 2000 (P00), *A&A*, 362, 169.
- Pribulla, T., Chochol, D., Heckert, P. A., Errico, L., Vittone, A. A., Parimucha, Š. & Teodorani, M., 2001 (P01a), *A&A*, 371, 997.
- Pribulla, T. & Vanko, M., 2002 (P02b), *CoSka*, 32, 79.
- Pribulla, T., Vanko, M., Chochol, D. & Parimucha, Š., 2001 (P01b), *CoSka*, 31, 26.
- Rahunen, T. & Vilhu, O., 1982, in *Binary and Multiple Stars as Tracers of Stellar Evolution*, eds Kopal, Z. & Rahe, J., *IAU Coll.*, 69, 289
- Rainger, P. P., Hilditch, R. W. & Bell, S. A., 1990 (R90a), *MNRAS*, 246, 42.
- Robertson, J. A & Eggleton, P. P., 1977, *MNRAS*, 179, 359.
- Rovithis, P., Rovithis-Livaniou, H. & Niarchos, P. G., 1990 (R90b), *A&AS*, 83, 41.
- Rovithis-Livaniou, H., Rovithis, P. & Bitzaraki, O., 1992, *Ap&SS*, 189, 237.

- Rucinski, S. M., 1986, in *Instrumentation and Research Programmes for Small Telescopes*, eds Hearnshaw, J. B. & Cottrell, P. L., IAU Symp., 118, 159.
- Rucinski, S. M. & Lu, W., 2000 (R00a), MNRAS, 315, 587.
- Rucinski, S. M., Lu, W. & Mochnacki, S. W., 2000 (R00b), AJ, 120, 1133.
- Rucinski, S. M., Lu, W., Capobianco, C. C., Mochnacki, S. W., Blake, R. M., Thomson, J. R., Oloza, W. & Stachowski, G., 2002 (R02), AJ, 124, 173.
- Rucinski, S. M., Lu, W. & Shi, J., 1993 (R93), AJ, 106, 1174.
- Rucinski, S. M. & Lu, W., 1999 (R99), AJ, 118, 2451
- Russo, G., Sollazzo, C., Maceroni, C. & Milano, L., 1982, A&AS, 47, 211
- Samec, R. G. & Hube, D. P., 1991 (S91), AJ, 102, 1171
- Samec, R. G., Pauley, B. R. & Carrigan, B. J., 1997 (S99), AJ, 113, 401
- Sanford, R. F., 1937 (S37), Mt Wilson Contr., 574, 195.
- Sarma, M. B. K., Vivekananda Rao, P. & Abhyankar, K. D., 1996, ApJ, 458, 371.
- Sarna, M. J. & Fedorova, A. V., 1989, A&A, 208, 111.
- Schou, J., Antia, H. M., Basu, S., Bogart, R. S., Bush, R. I., Chitre, S. M., Christensen-Dalsgaard, J., Di Mauro, M. P., Dziembowski, W. A., Eff-Darwich, A., Gough, D. O., Haber, D. A., Hoeksema, J. T., Howe, R., Korzennik, S. G., Kosovichev, A. G., Larsen, R. M., Pijpers, F. P., Scherrer, P. H., Sekii, T., Tarbell, T. D., Title, A. M., Thompson, M. J. & Toomre, J., 1998, ApJ, 505, 390.
- Ségransan, D., Delfosse, X., Forveille, T., Beuzit, J.-L., Udry, S., Perrier, C. & Mayor, M., 2000 (S00), A&A, 364, 665.
- Sezer, C., Gülmen, Ö. & Güdür, N., 1985 (S85), IBVS, 2743.
- Shaw, J. S., 1994, MSAI, 65, 95.
- Shaw, J. S., Guinan, E. F. & Garasi, C. J., 1990 (S90), BAAS, 22, 1296.
- Shaw, J. S., Guinan, E. F. & Ivester, A. H., 1991 (S91), BAAS, 23, 1415.
- Shaw, J. S., Caillault, J.-P. & Schmitt, J. H. M. M., 1996, ApJ, 461, 951.

- Shu, F. H., 1980, in *Close Binary Stars: Observations and Interpretation*, eds. Plavec, M. J., Popper, D. M. & Ulrich, R. K., IAU Symp., 88, 477.
- Shu, F. H. & Lubow, S. H., 1981, ARA&A, 19, 277.
- Shu, F. H., Lubow, S. H. & Anderson, L., 1976, ApJ, 209, 536.
- Shu, F. H., Lubow, S. H. & Anderson, L., 1979, ApJ, 229, 223.
- Shu, F. H., Lubow, S. H. & Anderson, L., 1980, ApJ, 239, 937.
- Smith, D. H., Robertson, J. A. & Smith, R. C., 1980, MNRAS, 190, 177.
- Thompson, M. J., Christensen-Dalsgaard, J., Miesch, M. S. & Toomre, J., 2003, ARA&A, 41, 599.
- Tutukov, A. V. & Yungel'son, L. R., 1971, Nauch. Info., 20, 86.
- van Hamme, W., Samec, R. G., Gothard, N. W., Wilson, R. E., Faulkner, D. R. & Branly, R. M., 2001 (vH01), AJ, 122, 3436.
- Vaňko, M., Pribulla, T., Chochol, D., Parimucha, Š., Kim, C. H., Lee, J. W. & Han, J. Y. 2001 (V01), CoSka, 31, 129.
- Vilhu, O. & Rahunen, T., 1981, in *Fundamental Problems in the Theory of Stellar Evolution*, ed. Sugimoto, D., Lamb, D. Q. & Schramm, D. N., IAU Symp., 93, 181.
- Vilhu, O., 1982, A&A, 109, 17.
- Webbink, R. F., 1976a, ApJ, 209, 829.
- Webbink, R. F., 1976b, ApJS, 32, 583.
- Webbink, R. F., 1977a, ApJ, 211, 486.
- Webbink, R. F., 1977b, ApJ, 211, 881.
- Webbink, R. F., 1977c, ApJ, 215, 851.
- Wolf, M., Molk, P., Hornoch, K. & Sarounov, L., 2000 (W00), A&AS, 147, 243”
- Yakut, K., Ulas, B. & Kalomeni, B. 2004 (Y04a), MNRAS, in press.
- Yakut, K., İbanoğlu, C., Kalomeni, B. & Değirmenci, Ö. L., 2003 (Y03a), A&A, 401, 1095.
- Yakut, K., Kalomeni, B. & İbanoğlu, C., 2004 (Y04b), A&A, 417, 725

- Yamasaki, A., Jugaku, J. & Seki, M., 1988 (Y88), *AJ*, 95, 894.
- Yang, Y. & Liu, Q., 1999 (Y99), *A&AS*, 136, 139.
- Yang, Y. & Liu, Q., 2003 (Y03b), *New Astronomy*, 8, 465.
- Yang, Y. & Liu, Q., 2003 (Y03c), *AJ*, 126, 1960.
- Yang, Y. & Qian, S., 2004 (Y04c), *PASP*, 116, 826.
- Yungel'son, L. R., 1971, *Nauch. Info.*, 20, 94.
- Yungel'son, L. R., 1972, *AZh*, 49, 1059.
- Zeilik, M., Gordon, S., Jaderlund, E., Ledlow, M., Summers, D. L., Heckert, P. A., Budding, E. & Banks, T. S., 1994 (Z94), *ApJ*, 421, 303.
- Zhang, X. B. & Zhang, R. X., 2004 (Z04), *MNRAS*, 307, 315.
- Zola, S., Niarchos, P. G., Manimanis, V. N. & Dapergolas, A., 2001 (Z01), *A&A*, 374, 164.
- Zola, S., Rucinski, S. M., Baran, A., Ogłóza, W., Pych, W., Kreiner, J. M., Stachowski, G., Gazeas, K., Niarchos, P., Siwak, M. & Drozd, M., 2004 (Z04), *ActaA*, 54, 299-312